1. Let the random variable $X$ be the number of women in a jury of size 12, selected "at random" from a pool of 10 women and 10 men.
   a. Find $P(X \leq 3)$.
      
      $0.00988$

b. Find $E(X)$.
   $12 \times 10/20 = 6$. 
2. Suppose that a login occurs on a computer on a particular 1 second interval with probability $p$, and there is either 0 or 1 login in each second. Furthermore, suppose that extensive data shows that the average time between logins is 10 seconds. 
   a. Give the probability mass function for the geometric random variable $T$ which is the time until the next login (in seconds).
   \[ P(T = k) = (1 - p)^{k-1} p, \quad k = 1, 2, \ldots \]

   b. Give a reasonable value for $p$ based on the extensive data and the mean of $T$.
   \[ p \approx 1/10 = .10. \]

   c. What is the distribution of the number of logins in 60 seconds? (Just use the concise "~" notation and use the value of the parameter $p$ from part b.)
   \[ N \sim \text{bin}(60, .10) \text{ or } N \sim \text{Poisson}(6). \]

   d. Numerically approximate the probability of 3 or more logins in 60 seconds.
   \[ P(N \geq 3) = 1 - P(N \leq 2) = .938. \]
3. Consider the following problem of reasoning about causes from effects. Top quality products from company A fail with probability \( p = 0.001 \), whereas copies from company B fail with probability \( p = 0.01 \). It is known that 80% of the market is supplied by company A, whereas 20% of the market is supplied by company B. Suppose two items are chosen randomly from a single box that came from one of the two companies. If they both fail, what is the conditional probability that the item came from company A?

\[
\frac{0.001^2 \times 0.80}{0.001^2 \times 0.80 + 0.01^2 \times 0.20} = 0.0385.
\]

4. A player receives winnings \( W = 10 \times (X + Y) \), where \( X, Y \), are the values on two dice tossed independently. Find \( E(W) \), the expected winnings, and the variance of \( W \) and give their units.

\[
E(W) = 10 \times \frac{252}{36} = \$70, \ Var(W) = E(W^2) - 70^2 = 10^2 \times \frac{1974}{36} - 70^2 = \$^2583.33.
\]
5. Suppose that teams A and B play one another in a best-of-5 tournament, and the teams are evenly matched. Find the expected number of games in the tournament.
   4.125 games

6. Suppose that I have password "bluedevils" in mind, and I know there is exactly one user with this password among the 5000 acpub users. I decide to go through the users in a random order until I get the right user and access his/her account (searching through users is faster than searching through passwords).
   a. Find the conditional probability of getting the right user on trial 2, given that I got the wrong user on trial 1.
      \[ \frac{1}{4999} \]
   
   b. Also, give the unconditional probability of getting the right user on or before trial 1000.
      \[ P(T \leq 1000) = \frac{1000}{5000} = \frac{1}{5}. \]

   c. Suppose now that there are two users with this password. Find the probability of getting either or both of them before trial 1000.
      \[ 1/5 + 1/5 - \left( \frac{1000 \times 999}{5000 \times 4999} \right) = 0.360032. \]