1. Let $U \sim \text{unif}(0,1)$, and suppose we want to generate a random variable that has the cdf $F(t) = 1 - e^{-t^3}$, $t > 0$, $F(t) = 0$, $t \leq 0$. What should we do to $U$ to get a random variable that has the density implied by the cdf $F$?

$$T = (-\log(1 - U))^{1/3} \sim F \text{ (Weibull)}.$$

2. Let the weekly revenue in a restaurant have the $N(2000, 200^2)$ distribution.

   a. What is the probability that the total revenue over two weeks is less than $3,600$?

   $$P(R_1 + R_2 < 3600) = P(Z < (3600 - 4000)/(\sqrt{2} \times 200)) = .0786.$$  

   b. What is the probability that the revenues in two weeks differ in absolute value by at least $200$?

   $$P(|R_1 - R_2| \geq 200) = 1 - P(|R_1 - R_2| < 200) = 1 = P(-1/\sqrt{2} < Z < 1/\sqrt{2}) = .4795.$$
3. Let $S$ be the value on the sum of four dice, thrown independently.
   a. What is $E(S)$?

   $E(S) = 14$.

   b. What is the variance of $S$?

   $\sigma^2 = 4 \times 35/12 = 35/3 = 11.667$.

   c. Approximate $P(S \geq 16)$, using a Normal approximation.

   $P(S \geq 16) \approx P(Z \geq \frac{15.5 - 14}{\sqrt{11.667}}) = .3303$. 
4. Suppose two random variables $X, Y$ have a joint density given by

$$f(x, y) = \frac{e^{-x}}{x}, \quad 0 < y < x < \infty$$

and $f(x, y) = 0$ elsewhere. This corresponds to choosing the pair $(X, Y)$ by waiting
an exponential amount of time for $X$, then choosing $Y$ uniformly from the interval
$(0, X)$.

a. Find the covariance of $X, Y$.

$Cov(X, Y) = E(XY) - E(X)E(Y) = 1 - 1 \times 1/2 = 1/2$.

b. Find the marginal density of $X$.

$f_X(x) = e^{-x}, \quad x > 0$ and $f(x) = 0, \quad x < 0$.

c. Find the conditional density $f_{X|Y}(x \mid y = 1)$ of $X$ given $Y = 1$.

$$f_{X|Y}(x \mid y = 1) = \begin{cases} 
0 & x < 1 \\
\frac{e^{-x}/x}{\int_1^\infty e^{-t}/t \, dt}, & x > 1.
\end{cases}$$
5. Suppose a pair of Normal random variable $(X, Y)$ have the property that $\sigma_x = 1.0, \sigma_y = 2.0, \rho = 1/2$. Find the standard deviation of $(X + Y)/2$, their average.

$$\sigma = \sqrt{2.0^2 + 1.0^2 + 2 \times .5 \times 2.0 \times 1.0/2} = 1.3229.$$  

6. Suppose the number of airplane accidents worldwide in one month has the Poisson(2) distribution. Find the probability of 5 or more accidents in the next three months.

$$N \sim \text{Poisson}(6), \ P(N \geq 5) = 1 - P(N \leq 4) = 1 - e^{-6}(115) = .7149.$$