Due: by 5pm Friday, November 8.

Total weight in final assessment: 4%.

1. Suppose that $X_1, X_2, \ldots, X_n$ are iid $N(0, 1)$. Let

$$Y_1 = \frac{1}{n} \sum_{i=1}^{n} |X_i|$$

$$Y_2 = \left| \frac{1}{n} \sum_{i=1}^{n} X_i \right|$$

Calculate

(a) $E(Y_1)$
(b) $E(Y_2)$

2. Suppose that $X_1, X_2, \ldots, X_6$ are iid $N(0, \sigma^2)$. Use the $X_i$’s to construct statistics with the following distributions:

(a) $\sigma^2 \chi^2_5$
(b) $t_5$
(c) $F_{2, 2}$

3. Suppose that $X_1, X_2, \ldots, X_n$ are iid with population pdf:

$$f(x) = \begin{cases} 
1/\theta & \text{if } 0 < x < \theta \\
0 & \text{otherwise}
\end{cases}$$

Calculate the pdf of $X_{(i)}$ for $i = 1, \ldots, n$.

4. Suppose that $X = (X_1, X_2, \ldots, X_m)$ is an iid sample of Bernoulli($p$) random variables, i.e.

$$P(X_i = 0) = 1 - p$$
$$P(X_i = 1) = p.$$
Suppose also that \( Y = (Y_1, Y_2, \ldots, Y_n) \) is an iid sample of Bernoulli\((1-p)\) random variables, i.e.

\[
P(Y_i = 0) = p \\
P(Y_i = 1) = 1 - p.
\]

Show that

\[
T(X, Y) = \sum_{i=1}^{m} X_i - \sum_{i=1}^{n} Y_i
\]

is a sufficient statistic for \( p \).

5. Suppose that \( X = (X_1, X_2, \ldots, X_n) \) is an iid sample with population pdf given by

\[
f(X_i | \mu, \lambda) = \begin{cases} 
\lambda \exp\{-\lambda(X_i - \mu)\} & \text{if } X_i > \mu \\
0 & \text{if } X_i \leq \mu
\end{cases}
\]

Show that

\[
T(X) = \left( X_{(1)}, \sum_{i=1}^{n} X_i \right)
\]

is a sufficient statistic for \( \mu \) and \( \lambda \).

6. Suppose that \( X \sim N(\mu, 1) \) Show that

\[
T(X) = |X|
\]

is not a sufficient statistic for \( \mu \).

7. Suppose that \( X = (X_1, X_2, \ldots, X_n) \) is an iid sample with population pdf \( f(x | \theta) \). Suppose also that \( T(X) \) is a sufficient statistic for \( \theta \) and \( T^*(X) \) is a statistic satisfying for all \( x \)

\[
T(x) = g(T^*(x))
\]

for some function \( g \). Show that \( T^*(X) \) is a sufficient statistic for \( \theta \).