1. This exam is CLOSED BOOK; however, you may use two 8.5” x 11” sheet (front and back) of notes.

2. With this exam you will need:
   
   (a) Tables of $z$, $t$, $F$
   
   (b) A calculator

3. You must show all steps to receive full credit. Answers given with no supporting work will receive no credit.

4. You have 3 hours to complete the exam; you must finish by the end of the period (12 noon).

NSEES Honor Code

I HAVE NOT GIVEN OR RECEIVED, NOR WILL I GIVE, INFORMATION BEFORE, DURING OR AFTER THIS EXAM RELATING TO ITS CONTENTS OR COMPLETION THAT WOULD CONFER AN ADVANTAGE OVER STUDENTS NOT PRIVY TO THIS INFORMATION, UNLESS SPECIFICALLY TOLD BY THE INSTRUCTOR THAT SUCH CONDUCT IS PERMISSIBLE. I HAVE NOT USED DESIGNATED STUDY MATERIALS FOR THIS COURSE IN A MANNER THAT WOULD PREVENT THEM FROM BEING EQUALLY AVAILABLE TO OTHER STUDENTS. I WILL REPORT ANY VIOLATIONS OF THIS CODE TO THE INSTRUCTOR IN CHARGE OF THE COURSE AND TO THE DEAN.

Signature: ___________________________  Date: ___________________________

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</table>
1. Is increased fish consumption related to lowered risk of coronary heart disease? 40 Dutchmen were interviewed to determine the typical number of grams of fish they consumed per day. Twenty years later, the level of dietary cholesterol (risk factor for heart disease) for each was recorded.

Let \( \mu_{\text{none}} \) = mean cholesterol level for men who do not eat fish. (n=18)

Let \( \mu_{\text{fish}} \) = mean cholesterol level for men who eat 45 grams/day of fish. (n=22)

The mean cholesterol of non-fish eaters (coded “N”) minus the mean cholesterol of fish eaters (“F”) is calculated and the test statistic is formed. Results of two-sided two sample t-test of the claim that \( \mu_{\text{none}} \neq \mu_{\text{fish}} \) are summarized below.

**Standard Two-Sample t-Test**

data:  N and F

t = -2.02, df = xxx, p-value = xxx

alternative hypothesis: true difference in means is not equal to 0

**sample estimates:**

<table>
<thead>
<tr>
<th></th>
<th>mean of N</th>
<th>mean of F</th>
</tr>
</thead>
<tbody>
<tr>
<td>138</td>
<td>142</td>
<td></td>
</tr>
</tbody>
</table>

(a) What is the two-sided p-value from the above output? [5pts]

(b) What is the one-sided p-value for a test of the claim that cholesterol levels are higher for men who do not eat fish than for men who eat 45 grams per day? Show work. [10pts]

(c) What assumptions are necessary for the validity of the p-value you calculated in (1a)? [6pts]

(d) In terms of the problem at hand, give one possible way that correlation among the observed cholesterol levels could occur. [4pts]
(e) **True or False. Circle one.** The $p$-value is always dependent on sample size. [5pts]

(f) **True or False. Circle one.** Because this was not a randomized or experimental study, we cannot generalize our results to the population of Dutchmen. [5pts]

2. Does dust extract induce respiratory disease in susceptible cotton farm workers? A study is planned for a sample of farm workers. A blood sample will be taken from each farm worker, and portions will be incubated with two concentrations of dust extract. Treatment A is 0 mg/ml dust extract and Treatment B is 0.2 mg/ml dust extract. A measurement of cell activity, called cyclic AMP level, or cAMP, will be taken for each dust extract level. The units of cAMP are picomoles per million cells.

(a) Is blocking employed in this experiment? Indicate yes or no and explain. [5pts]

(b) We wish to evaluate the claim that cell activity (mean cAMP level) is lower for Treatment B than in Treatment A. Write out the hypotheses of interest, and give the parametric test used and the assumptions required. [10pts]

(c) What does a Type II error mean in the context of the claim in (2b)? Give your answer in terms that a cotton farm manager could understand. [5pts]
(d) Another group of researchers would like to evaluate the claim in (2b) using a non-parametric test that reflects the relative magnitudes of the differences between A and B. What test would they use and what are required assumptions? [6pts]

(e) The alternative hypothesis of particular interest is that the difference in mean cell activity for the 0 mg/ml concentration group (A) exceeds the 0.2 mg/ml concentration group (B) by 5 pmol/million cells. [8pts]

The researchers consider decreasing $\alpha$ by 50%. Fill in the letter of your response in the blanks.

The Type I error will ________ and the Type II error will ________

(a) decrease
(b) stay the same
(c) increase

(f) The figure below gives the sampling distribution of the difference in means (A-B) under the null hypothesis (solid line) and under the alternative hypothesis (dotted line) described in (2c).

![Graph showing normal distribution](image)

The rejection region considered is that the difference in sample means (A-B) exceeds 4. **Shade and clearly label** the areas that represent the Type I error and the Type II error for the test. [6pts]
3. Suppose that each of two different varieties of corn is treated with two different types of fertilizer in order to compare the yields, and that 10 independent replications are obtained for each of the four variety/fertilizer combinations.
Let \( X_{ijk} \) denote the yield on the \( k \)th replication of the combination of variety \( i \) with fertilizer \( j \) \((i = 1, 2; j = 1, 2; k = 1, \cdots, 10)\). Assume that the assumptions of a two-way ANOVA are met. We denote the population mean of each variety/fertilizer combination as \( \mu_{ij} \).

(a) Researcher A appropriately considers a one-way ANOVA for the data. She creates 4 groups: Variety 1/Fertilizer 1, Variety 1/Fertilizer 2, Variety 2/Fertilizer 1, Variety 2/Fertilizer 2. Give the degrees of freedom for the mean squared error (MSE) for her analysis. \([10\text{pts}]\)

(b) Researcher A uses her model to test whether the difference in mean yields between the two fertilizers for variety 1 is the same as the difference in mean yields between the two fertilizers for variety 2. Give the hypotheses tested, and the sampling distribution (with associated degrees of freedom) that will be used to obtain a p-value. \([10\text{pts}]\)

(c) Researcher B appropriately considers an additive model and computes a 2-way ANOVA for variety and fertilizer effects. Will the degrees of freedom for the mean squared error (MSE) be different from your answer in 3a for his analysis? If yes, give the degrees of freedom. If no, explain why. \([5\text{pts}]\)

(d) Write out Researcher B’s model. \([5\text{pts}]\)
(e) Researcher B uses his model to test whether the effect of fertilizer for variety 1 is the same as the differences in yield between the two fertilizers for variety 2. Give the hypotheses tested, and the sampling distribution (with associated degrees of freedom) that will be used to obtain a p-value. [10pts]

(f) True or False. Circle one. If a non-additive model is fit to these data, the total degrees of freedom would not change. [5pts]

4. An automobile manufacturer wishes to study the effects of differences between drivers (factor A) and differences between car colors (factor B) on gasoline consumption. The manager selected four colleagues as drivers. Five cars of the same model but different colors were randomly selected from the assembly line. Each driver drove each car twice over a 40-mile test course and the miles per gallon were recorded.

(a) Which of the following is true about this design? [10pts] Circle all that apply.
   i. It is a completely randomized design.
   ii. It is a balanced design.
   iii. It is a saturated design.
   iv. It employs replication.
   v. It is a complete design.

(b) The manufacturer uses fuel consumption as a way to measure speed of the car, reasoning that cars that drive faster use more gasoline. Assume for the moment that this is true. A non-additive model is fit, which in Splus is written as

\[ gas \sim driver + color + driver:color \]  \hspace{1cm} (1)

He evaluates the significance of the interaction term.

Based on an ANOVA table, the sum of squares for \texttt{driver:color} is equal to 2.44 and a sum of squares for \texttt{Residuals} is equal to 3.52. Give the \textit{p-value} and write, in language a car salesman could understand, the meaning of this \textit{p-value}. [10pts]
(c) The auto manufacturer now considers an additive model for the data, producing the following partial output:

<table>
<thead>
<tr>
<th></th>
<th>Df</th>
<th>Sum of Sq</th>
<th>Mean Sq</th>
<th>F Value</th>
<th>Pr(&gt;F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>driver</td>
<td>3</td>
<td>280.28</td>
<td>93.43</td>
<td>xxxxx</td>
<td>xxxxx</td>
</tr>
<tr>
<td>auto</td>
<td>4</td>
<td>94.71</td>
<td>23.68</td>
<td>xxxxx</td>
<td>xxxxx</td>
</tr>
<tr>
<td>Residuals</td>
<td>x</td>
<td>5.96</td>
<td></td>
<td></td>
<td>xxxxx</td>
</tr>
</tbody>
</table>

After calculating the overall mean to be 30.05, he produces a table of effects:

Tables of effects

<table>
<thead>
<tr>
<th></th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
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<tbody>
<tr>
<td>driver</td>
<td>-3.117</td>
<td>4.102</td>
<td>-1.198</td>
<td>0.2125</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th></th>
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<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>auto</td>
<td>-1.085</td>
<td>2.202</td>
<td>-2.135</td>
<td>1.115</td>
<td>-0.0975</td>
</tr>
</tbody>
</table>

Assume for this part only that the four drivers represent different skill levels, and that the driver effect can be considered a blocking variable. Calculate a 95% confidence interval for the difference in gasoline consumption between auto 2 (red) and auto 3 (pink). [10pts]

(d) Now assume that the manufacturer has selected the four drivers at random, and that the five cars are a random sample of the same model, year, and color. We are no longer interested in specific effects of particular drivers or cars, but we are interested in the relative magnitudes of the variance components for the set of cars and the set of drivers.

i. We will test whether there is significant variation among drivers relative to cars with respect to gas consumption. Write out a model to answer this question and define all parameters. (Define all of your indices.) [10pts]
ii. Provide hypotheses, test statistic, p-value. Compare your p-value to $\alpha = 0.05$. State the conclusion of your test in terms of the problem. [10pts]

iii. Estimate the components of variance as well as the intra-class correlation. Which factor has a greater effect on gasoline consumption? [6pts]

iv. True or False. Circle one. For the model considered in problem 4(d)i, as the number of drivers increases, the model contains more parameters, and thus, the driver effect has more degrees of freedom. [4pts]
5. Consider the following hypothetical data on incidents regarding the Space Shuttle O-ring as a function of temperature. Which of the following tests would be appropriate for these data?

<table>
<thead>
<tr>
<th>Launch Temperature</th>
<th>Number of O-Ring Incidents</th>
</tr>
</thead>
<tbody>
<tr>
<td>Below 65°</td>
<td>1 1 1 1 1 1 1 2 2 2 2 2 2</td>
</tr>
<tr>
<td>Above 65°</td>
<td>0 0 0 0 0 0 0 0 0 0 0 0 1 1 3</td>
</tr>
</tbody>
</table>

[5pts]
(a) Wilcoxon rank sum test  
(b) Wilcoxon signed rank test  
(c) Permutation test  
(d) Two-sample t-test  
(e) Sign test
6. There is concern that plasma testosterone levels may be affected by marijuana use. Plasma testosterone levels were measured (ng/100ml) for a group of young males who used marijuana to different degrees. Data are summarized below:

<table>
<thead>
<tr>
<th>Summary Statistics</th>
<th>Control (0 joints/week)</th>
<th>Mild Users (5-9 joints/week)</th>
<th>Heavy Users (≥ 10 joints/week)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X$</td>
<td>742</td>
<td>503</td>
<td>309</td>
</tr>
<tr>
<td>$n$</td>
<td>20</td>
<td>11</td>
<td>9</td>
</tr>
</tbody>
</table>

For these data $s_{pooled} = 300$.

The authors calculated confidence intervals for various contrasts to find significant results, and found that the most significant one was $\gamma = \mu_{control} - 1/2(\mu_{mild} + \mu_{heavy})$. Explain what this contrast means and calculate a confidence interval using the appropriate multiple comparison procedure. Give a one sentence interpretation of the confidence interval. [15pts]

7. In the process of model selection, a researcher finds that the model that has the highest $R^2$ is different from the model with the lowest $BIC$. How could this happen if there is no mistake in calculations? [10pts]
8. Samples of fish were taken from each of four lakes and analyzed for PCB concentration (in ppm). We are interested in whether the data provide sufficient evidence to indicate differences in mean PCB content for the four lakes.

The lakes are 2 km, 4 km, 5 km, and 9 km from a plant that produces PCBs. Assume that PCB levels in fish decrease with increasing distance from the plant. No further information is available.

(a) Draw a boxplot of the PCB levels in the 5 regions that suggests the need for a reciprocal transform. [5pts]

(b) An analyst performed a 1-way ANOVA on the data without performing the needed reciprocal transform. Draw a clearly labeled residual plot for this situation. [5pts]

(c) Another analyst performed a 1-way ANOVA on a similar dataset, and found that the MSE was zero. Draw a clearly labeled residual plot for this situation. [5pts]