Solution for HW1

STA113 ISDS

September 5, 2004

§13.

a. \( A_1 \cup A_2 = \{\text{awarded project 1 or 2}\} \)
\[
P(A_1 \cup A_2) = P(A_1) + P(A_2) - P(A_1 \cap A_2)
\]
\[
= 0.22 + 0.25 - 0.11 = 0.36
\]

b. \( A'_1 \cap A'_2 = \{\text{awarded neither project 1 nor 2}\} \)
\[
P(A'_1 \cap A'_2) = P((A_1 \cup A_2)') = 1 - P(A_1 \cup A_2) = 0.64
\]

c. \( A_1 \cup A_2 \cup A_3 = \{\text{awarded at least one project from 1, 2, and 3}\} \)
\[
P(A_1 \cup A_2 \cup A_3) = P(A_1) + P(A_2) + P(A_3) - P(A_1 \cap A_2) - P(A_1 \cap A_3) - P(A_2 \cap A_3) + P(A_1 \cap A_2 \cap A_3)
\]
\[
= 0.53
\]

d. \( A'_1 \cap A'_2 \cap A'_3 = \{\text{awarded none of 1, 2, 3}\} \)
\[
P(A'_1 \cap A'_2 \cap A'_3) = P((A_1 \cup A_2 \cup A_3)') = 1 - 0.53 = 0.47
\]
e. $A'_1 \cap A'_2 \cap A_3 = \{\text{only awarded project 3}\}$

$$P(A'_1 \cap A'_2 \cap A_3) = 0.17$$

Hint: $P(A'_1 \cap A'_2 \cap A_3) + P(A'_1 \cap A'_2 \cap A'_3) = P(A'_1 \cap A'_2)$

f. $(A'_1 \cap A'_2) \cup A_3$

$= \{\text{awarded project 3, or awarded neither project 1 nor 2}\}$

$$P((A'_1 \cap A'_2) \cup A_3) = P(A'_1 \cap A'_2) + P(A_3) - P(A'_1 \cap A'_2 \cap A_3) = 0.75$$

§26.

a. $P(A'_1) = 1 - P(A_1) = 0.88$

b. $P(A_1 \cap A_2) = P(A_1) + P(A_2) - P(A_1 \cup A_2) = 0.06$

c. $P(A_1 \cap A_2 \cap A'_3) = P(A_1 \cap A_2) - P(A_1 \cap A_2 \cap A_3) = 0.05$

d. $1 - P(A_1 \cap A_2 \cap A_3) = 0.99$

Hint: the event that the system has all of these defects and the event that the system had at most two of these defects are mutually exclusive.

§30.

a: $P_{3,8} = 336$

b: $N = \binom{30}{6} = 593775$

c: $n = \binom{8}{2} \binom{10}{2} \binom{12}{2} = 83160$
d: \( P = n/N = 0.1400 \)

e: \( \frac{{8 \choose 6} + {10 \choose 6} + {12 \choose 6}}{N} = 0.00196 \)

§33.

a: \( N = \binom{25}{5} = 53,130 \)

b: \( n = \binom{8}{4} \binom{17}{1} = 1,190 \)

c: \( P = \frac{n}{N} = \frac{1190}{53130} = 0.022 \)

d: \( P = \frac{{8 \choose 4} {17 \choose 1} + {8 \choose 5}}{N} = 0.023 \)

*Hint:* At least 4 having visible cracks means 4 or 5 of those selected having cracks

§40.

a: \( n = \frac{P_{12,12}}{(P_{3,3})^4} = 369,600 \)

*Hint:* if the three A’s, B’s, C’s and D’s were distinguished from one another, then the answer is \( n = p_{12,12} = 479,001,600 \) when the subscripts are removed from A’s. the number of chain molecule should be divided by \( p_{3,3} \). So, the answer of this
question is \( n = \frac{P_{12,12}}{(P_{3,3})^4} \)

b: \( P(\text{All three molecules of each type end up next to one another}) = \frac{p_{4,4}}{n} \approx 6.4935 \times 0.05 \)

*Hint:* We can suppose the three molecules of the same type to be 1 molecule, (such as BBBAAADDCCC=BADC), then the number of the chain molecule described in the question is \( P_{4,4} \).

\[ \text{§48.} \]

\( a: \) \( P(A_2|A_1) = \frac{P(A_1 \cap A_2)}{P(A_1)} = \frac{0.06}{0.12} = 0.5 \)

\( b: \) \( P(A_1 \cap A_2 \cap A_3|A_1) = \frac{P(A_1 \cap A_2 \cap A_3)}{P(A_1)} = \frac{1}{12} \)

\( c: \) \( P((A_1 \cap A_2^c \cap A_3^c) \cup (A_1^c \cap A_2 \cap A_3^c) \cup (A_1^c \cap A_2^c \cap A_3))|A_1 \cup A_2 \cup A_3) = \frac{P((A_1 \cap A_2^c \cap A_3^c) \cup (A_1^c \cap A_2 \cap A_3^c)) + P((A_1^c \cap A_2 \cap A_3^c)) + P((A_1^c \cap A_2^c \cap A_3))}{P(A_1 \cup A_2 \cup A_3)} = 0.05/0.14 = 0.3571 \)

\( d: \) \( P(A_3^c|A_1 \cap A_2) = \frac{P(A_3^c \cap A_1 \cap A_2)}{P(A_1 \cap A_2)} = \frac{0.05}{0.06} = 0.8333 \)

*Hint:* Draw a venn diagram

\[ \text{§59.} \]
a: \(P(A_2 \cap B) = P(B|A_2)P(A_2) = 0.21\)

b: \(P(B) = P(B|A_1)P(A_1) + P(B|A_2)P(A_2) + P(B|A_3)P(A_3) = 0.455\)

c: \(P(A_1|B) = \frac{P(B|A_1)P(A_1)}{\sum_{i=1}^{3} P(B|A_i)P(A_i)} = 0.2637\)
\(P(A_2|B) = \frac{P(B|A_2)P(A_2)}{\sum_{i=1}^{3} P(B|A_i)P(A_i)} = 0.4615\)
\(P(A_3|B) = \frac{P(B|A_3)P(A_3)}{\sum_{i=1}^{3} P(B|A_i)P(A_i)} = 0.2747\)

§61.

Let \(A_i = \{i \text{ defective components in the batch}\}, \text{ for } i = 0, 1, 2\)
\(B_a = \{\text{neither tested component is defective}\}\)
\(B_b = \{\text{One of the tested components is defective}\}\)

a: \(P(A_0|B_a) = \frac{1 \times 0.5}{1 \times 0.5 + \binom{9}{2}/\binom{10}{2} \times 0.3 + \binom{8}{2}/\binom{10}{2} \times 0.2} = 0.5784\)
\(P(A_1|B_a) = 0.2776\)
\(P(A_2|B_a) = 0.1439\)

Hint: for \(i = 0, 1, 2\)
\(P(A_i|B_a) = \frac{P(B_a|A_i)P(A_i)}{\sum_{j=0}^{2} P(B_a|A_j)P(A_j)}\)

b: \(P(A_0|B_b) = 0\)
\(P(A_1|B_b) = \frac{\binom{9}{1}/\binom{10}{2} \times 0.3}{\binom{9}{1}/\binom{10}{2} \times 0.3 + \binom{8}{2}/\binom{10}{2} \times 0.2} = 0.5784\)
\[ \begin{align*}
0.2 \times 0.3 &= 0.6 \\
0.2 \times 0.3 + 0.3556 \times 0.2 &= 0.4576 \\
P(A_2|B_b) &= 0.5424 \\

\text{Hint: for } i = 0, 1, 2 \\
P(A_i|B_b) &= \frac{P(B_b|A_i)P(A_i)}{\sum_{j=0}^{2} P(B_b|A_j)P(A_j)}
\end{align*} \]

§69.

a: \[ P(B'|A') = P(B') = 0.3 \]

\text{Hint: A and B are independent events}

b: \[ P(A \cup B) = P(A) + P(B) - P(A)P(B) = 0.82 \]

c: \[ P(A \cap B'|A \cup B) = \frac{P((A \cap B') \cap (A \cup B))}{P(A \cup B)} = \frac{0.12}{0.82} = 0.1463 \]

§78.

let event \( S_i = \{ \text{subsystem i works} \} \) for \( i = 1, 2 \) and \( C_i = \{ \text{component i works} \} \) for \( i = 1, 2, 3, 4 \)

\[ P(\text{system works}) = 1 - P(S_1' \cap S_2') = 1 - P(S_1')P(S_2') = 1 - P(C_1' \cap C_2')P(1 - C_3 \cap C_4) = 1 - 0.1^2 \times (1 - 0.9^2) = 0.9981 \]

\text{Hint: } S_1' \text{ and } S_2' \text{ are independent events.}

§80.

Notice \( P(A) = 1/6, P(B) = 1/6, P(c) = 1/6 \)
a: A and B are independent events;

b: B and C are independent events;
   Proof: \( P(B \cap C) = P(A \cap B) = \frac{1}{36} = P(B)P(C) \)

c: A and C are independent events;
   Proof: \( P(A \cap C) = P(A \cap B) = \frac{1}{36} = P(A)P(C) \)

d: A, B and C aren’t independent events;
   Proof: \( P(A \cap B \cap C) = \frac{1}{36} \neq \frac{1}{216} = P(A)P(B)P(C) \)