MTH135/STA104: Probability

Homework # 4 Due: Tuesday, Sep 27, 2005

Prof. Robert Wolpert

1. (from Prob 2 *p.* 121) Find Poisson approximations to the proabilities of the following events in 1000 independent trials with probability 0.005 of success on each trial:

a) 1 success;

b) 2 or fewer successes;

c) more than 3 successes.

2. (Prob 6 *p.* 122) A box contains 1000 balls, of which 2 are black and the rest white.

a) Which of the following is more likely to happen in 1000 draws with replacement from the box?
Fewer than 2 black balls Exactly 2 black balls More than 2 black balls

b) If two series of 1000 draws are made at random from this box, what approximately is the chance they produce the same number of black balls?

3. (from Prob 9, *p.* 122) A cereal company advertises a prize in every box of their cereal. In fact, only about 98% of the boxes have prizes. If a family buys one box of this cereal every week for a year, estimate the chance that they will collect at least 50 prizes. What assumptions are you making?

4. (from Prob 1, *p.* 127) Suppose you take a random sample of 9 tickets from a box containing 20 blue tickets and 30 white tickets.

a) What is the chance of getting exactly 4 blue tickets, if we sample *without* replacement?

b) What is the chance of getting exactly 4 blue tickets, if we sample *with* replacement?

5. (from Prob 3, *p.* 128) A deck of 52 cards is well shuffled and dealt to four players, each receiving 13 cards. Find:

a) The probability that all the aces are in the same hand

b) Two of the Jacks in a standard deck (J♣ and J♢) have both eyes visible, while the other two (J♠ and J♡) are shown in profile revealing on a single eye. Find the conditional probability that the first player holds both one-eyed jacks, given that she holds the (one-eyed) jack of spades.

c) Find the conditional probability that the first player holds both one-eyed jacks, given that she holds at least one of them

d) Find the probability that the same player holds both one-eyed jacks *and* all the spaces.

6. (from Prob 5, *p.* 128) Suppose 60% of a LARGE population of voters favor candidate Z. How large a sample must be taken for there to be a 99% chance that the majority of voters in the sample will favor candidate Z?

7. (from Prob 8, *p.*. 128) In a raffle with 100 tickets, five people each buy 20 tickets each. If there are 4 winning tickets in all, find the probability that:

a) The same person gets all 4 winning tickets

b) There are four different winners

8. (from Prob 13, *p.* 129) A factory which produces chips in lots of ten thousand uses the following scheme to check the quality of its product. From each lot a random sample of size 100 is taken. If the sample contains no more than two defectives, the lot is passed. If the sample contains more than two defectives, another random sample of size 100 is taken. If this sample contains no more than 2 defectives, the lot is passed. A lot that fails both tests is rejected. If a lot actually contains 500 (5%) defectives, find the chance it will pass the inspection? Approximate by sampling with replacement, and use the Poisson approximation.

9. (from Prob 22, *p.*. 134) Suppose that, on average, 4% of the purchasers of airline tickets do not appear for the departure of their flight. Determine how many tickets should be sold for a flight on an airplane with 400 seats, so that with probability 95% everyone who appears for the departure will have a seat. What assumptions are you making?

10. (from Prob 33, *p.* 136)

a) How can you simulate drawing a random number equally likely to be any of the numbers $\{1, 2, 3, 4, 5\}$, using only a fair six-sided die?

b) How can you simulate flipping a fair coin, using only a *possibly biased* six-sided die whose probability of showing face i satisfies $0 < p_i < 1$ for each $i \in \{1, 2, 3, 4, 5, 6\}$ and of course $\sum p_i = 1$, but the face probabilities are otherwise unknown?