

# MTH135/STA104: Probability

Homework # 9

Due: Tuesday, Nov 15, 2005

Prof. Robert Wolpert

1. Let  $X_1 \sim \text{Un}(0, 1)$  and  $X_2 \sim \text{Un}(0, 2)$  be independent, and let  $Y = X_1 + X_2$ . Find:

a) The probability  $P[Y \leq 2] = \underline{\hspace{10cm}}$

b) The density function for  $Y$ ,  $f(y) = \underline{\hspace{10cm}}$

c) The c.d.f. for  $Y$ ,  $F(y) = \underline{\hspace{10cm}}$

2. An auto parts customer must wait in one line to get the part she needs, then wait in another line to pay for the part to complete her purchase. The waiting time in the two lines are independent exponentially-distributed random variables with rates  $\alpha$  and  $\beta$ , respectively.

a) Find the density function for the total wait time.

3. Let  $X$  be the number on a fair six-sided die, and let  $Y \sim \text{Un}(0, 1)$  be drawn from the standard uniform distribution; set  $Z = X - Y$ .

a) Find the CDF for  $Z$ .

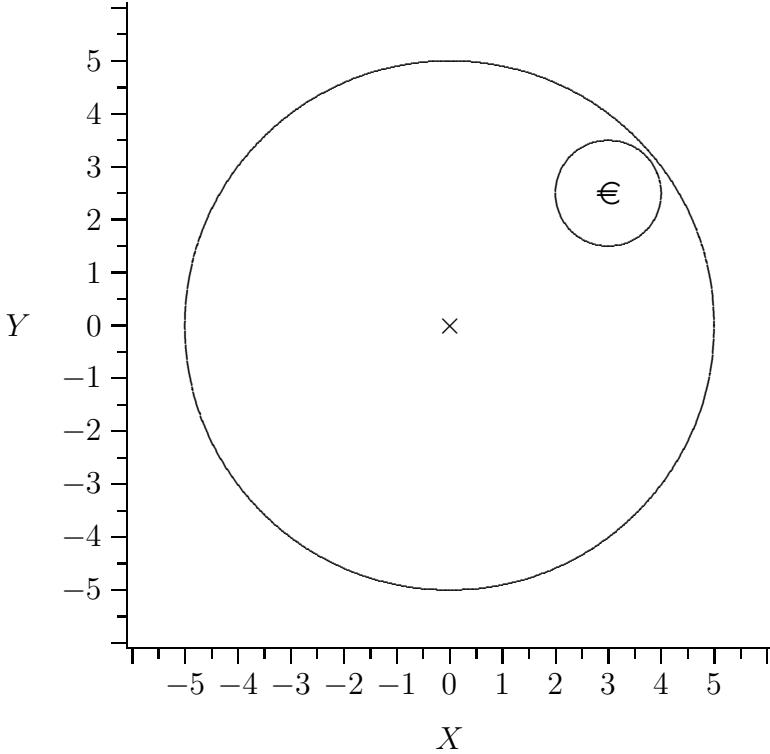
b) Does  $Z$  have a density function? If so, find it and identify the distribution. If not, explain why.

4. Let  $X \sim \text{No}(1, 1)$  have a normal distribution with mean  $\mu = 1$  and variance  $\sigma^2 = 1$ . Find:

a) The pdf for  $X$ ,  $f_X(x) = \underline{\hspace{10cm}}$

b) The pdf for  $Y = |X|$ ,  $f_Y(y) = \underline{\hspace{10cm}}$

- c) The pdf for  $V = (X - 1)^2$ ,  $f_V(v) = \underline{\hspace{2cm}}$
5. Chris takes an amount of time (in hours)  $U \sim \text{Un}(0, 4)$  to complete probability homework assignments, while Ryan takes an amount of time  $X \sim \text{Ex}(0.50)$ . Both have mean two hours (*i.e.*,  $\mathbb{E}U = 2 = \mathbb{E}X$ ).
- If they start at the same time, what is the probability that Chris finishes first?
  - Find the density function for the total time  $S = U + X$  they spend.
6. A Euro coin, with a diameter of 2cm, is dropped into an empty cat food can, whose round bottom has a diameter of 10cm.



- What is the probability that the center of the can bottom is covered by the coin?
- How many one-Euro coins would you have to drop to have at least a 50% chance of covering the center of the can bottom?
- What modeling assumptions did you make to answer parts a) and b) above?