

Final Examination

Mth 136 = Sta 114

Thursday, 2009 December 10, 2:00 – 5:00pm

- This is a **closed book** exam— please put your books on the floor.
- You may use a calculator and **two pages** of your own notes.
Do not share calculators or notes.
- Please ask me questions if a problem needs clarification.
- **Show your work.** **Boxing answers** helps me find them.
- Numerical answers: **four significant digits** or fractions in lowest terms. **Simplify** all expressions for full credit.
- Distribution and pdf/pmf tables are attached.

Cheating on exams is a breach of trust with classmates and faculty that will not be tolerated. After completing the exam please acknowledge that you have adhered to the Duke Community Standard:

- I will not lie, cheat, or steal in my academic endeavors;
- I will conduct myself honorably in all my endeavors; and
- I will act if the Standard is compromised.

Signature: _____

Print Name: _____

1.	/20	4.	/20
2.	/40	5.	/40
3.	/40	6.	/40
Total:			/200

Problem 1: For each part below, circle True or False and explain *why*:

a) (4) T F The Gamma distribution is conjugate for the Poisson distribution.

b) (4) T F If X has a χ^2_ν distribution then $\mathbb{E}X = \nu$.

c) (4) T F Every Maximum Likelihood Estimator is unbiased.

d) (4) T F For the same sample size n , a test of smaller size α will have higher power.

e) (4) T F Uniformly-most-powerful tests do not typically exist for two-sided hypothesis tests.

Problem 2: For 4pt each, circle “T” for True or “F” for False, or write short answers in the boxes. **Very very briefly**, *why*?

a) A P -value is the probability that H_0 is true. T F

b) What prior distribution is conjugate for Binomial data?

c) If a P -value satisfies $P < 0.01$ then Reject H_0 at $\alpha = 0.02$ T F

d) If $\{X_i\} \sim \text{Be}(\alpha, 1)$ then \bar{X} is sufficient for α . T F

e) The *power* of a test is the probability H_0 is rejected. T F

The remaining questions all concern a random sample $\mathbf{x} = \{X_i\}_{i \leq N}$ from the $\text{No}(\mu, \sigma^2)$ distribution.

f) The Likelihood Ratio Test (LRT) of the hypothesis $H_0 : \mu \leq 0$ vs. $H_1 : \mu > 0$ is UMP. T F

g) If $\sigma^2 = 32$, $N = 4$, $\bar{X} = 3$ with improper prior $\pi(\mu) = 1$ then what is the posterior distribution $\pi(\mu | \mathbf{x})$ for μ ?

h) Use a χ^2 dist'n to test $H_0 : \sigma^2 = 1$ against $H_1 : \sigma^2 > 1$. T F

i) Use a t distribution to estimate μ if $\sigma^2 = 1$ and $N \leq 10$. T F

j) What is the distribution of $\sum_{i \leq N} \frac{(X_i - \bar{X})^2}{\sigma^2}$ if $N = 10$?

Problem 3: You observe random samples from two normal distributions:

$$X_1, \dots, X_n \stackrel{\text{iid}}{\sim} \mathcal{N}(\mu_1, \sigma_1^2) \quad Y_1, \dots, Y_m \stackrel{\text{iid}}{\sim} \mathcal{N}(\mu_2, \sigma_2^2)$$

- a) (8) Give a null and alternate hypothesis for testing whether or not the two means are the same, when we know $\sigma_1 = \sigma_2 = 10$:
 - b) (8) Give a test statistic for testing these hypotheses, and give its probability distribution if the null hypothesis is true:
 - c) (4) Express the P -value for this test in terms of your test statistic and its distribution function.

Problem 3 (cont):

Still: $X_1, \dots, X_n \stackrel{\text{iid}}{\sim} \text{No}(\mu_1, \sigma_1^2)$ $Y_1, \dots, Y_m \stackrel{\text{iid}}{\sim} \text{No}(\mu_2, \sigma_2^2)$

d) (8) Give a null and alternate hypothesis for testing whether or not the two means are the same, when we know that $\sigma_1 = \sigma_2 = \sigma$ but we do *not* know the common value of σ

e) (8) Give a test statistic for testing these hypotheses, and give its probability distribution if the null hypothesis is true:

f) (4) Express the P -value for this test in terms of your test statistic and its distribution function.

Problem 4: In 1898 the Russian economist (of Polish descent) Ladislaus Bortkiewicz published the first account of the Poisson Distribution. For one of his applications he presented data showing the number of annual reported deaths due to horse kicks for each of 14 Prussian cavalry corps over a 20-year period. I kid you not. Here is a table of all the $20 \times 14 = 280$ observations:

Values:	0	1	2	3	4
Counts:	144	91	32	11	2

The total number of events was $S = 0 \cdot 144 + 1 \cdot 91 + 2 \cdot 32 + 3 \cdot 11 + 4 \cdot 2 = 168$, so the mean was $168/280 = 0.70$; since more than half (144 of 280) were zero, the median was 0. The sample SD is 0.8733407.

- a) (10) How can you test the hypothesis that these data come from a $\text{Po}(0.75)$ distribution? Give the hypothesis, an expression for the test statistic (simplify!), and its distribution if H_0 is true.

Problem 4 (cont):

- b) (10) How can you test the hypothesis that these data come from some $\text{Po}(\lambda)$ distribution for some unspecified $\lambda \geq 0$? Give the hypothesis, a test statistic (give the details), and its distribution if H_0 is true. Don't compute the numerical value of the test statistic or the P -value.

Problem 5: In a more abstract problem we have $N = 100$ independent observations $\mathbf{x} = \{X_i\}_{i \leq N}$, all from a $\text{Po}(\lambda)$ distribution with λ unknown. Some statistics from the data include

$$S(\mathbf{x}) = \sum_{i \leq N} X_i \quad T(\mathbf{x}) = \sum_{i \leq N} (X_i)^2 \quad U(\mathbf{x}) = \max_{i \leq N} X_i \quad V(\mathbf{x}) = \prod_{i \leq N} X_i!$$

- a) (10) Give the likelihood function $f(\mathbf{x} | \lambda)$ and derive the MLE $\hat{\lambda}(\mathbf{x})$.

Problem 5 (cont):

- b) (15) Construct an approximate 90% confidence interval for λ , using the Central Limit Theorem. What is the *exact* distribution of S ? What is its *approximate* distribution, by the Central Limit Theorem?

Problem 5 (cont):

- c) (15) Explain how to construct an (exact) 90% Bayesian interval for λ . With little or no experience with similar data, what prior distribution $\pi(\lambda)$ would you use? Be specific. What will the posterior distribution $\pi(\lambda | \mathbf{x})$ be?

Problem 6: Swoozie and Biff have decided to use a gamma prior distribution

$$\pi(\lambda) \sim \text{Ga}(\alpha, \beta)$$

for the rate parameter λ in a problem whose data have the exponential distribution

$$\mathbf{x} = \{X_i\} \stackrel{\text{iid}}{\sim} \text{Ex}(\lambda)$$

with $n > 2$ observations. By the way, they happen to know¹ the moments of the Gamma distribution, and they know that \bar{X} is sufficient for the exponential distribution, so they wrote down its value \bar{x} . All answers below will depend on one or more of α , β , n , and \bar{x} .

- a) (10) What is the posterior mean of λ ?

$$\mathbb{E}_\pi[\lambda | \mathbf{x}] = \underline{\hspace{2cm}}$$

- b) (5) Find the conditional expectation of the next ($n + 1^{st}$) observation, for this prior and data:

$$\mathbb{E}_\pi[X_{n+1} | \mathbf{x}] = \underline{\hspace{2cm}}$$

¹Recall that if $Y \sim \text{Ga}(\alpha, \beta)$ then $\mathbb{E}[Y^p] = \frac{\Gamma(\alpha+p)}{\Gamma(\alpha)} \beta^{-p}$ for any $p \in (-\alpha, \infty)$, even for $p < 0$; this follows from the integral $\int_0^\infty x^{\alpha-1} e^{-bx} dx = \Gamma(a)b^{-a}$ for any $a > 0$ and $b > 0$.

Problem 6 (cont):

Recall $\{X_i\}_{i \leq n} \stackrel{\text{iid}}{\sim} \text{Ex}(\lambda)$.

- c) (5) Find the MLE for the n observations. Show your work.

$$\hat{\lambda}_n(\mathbf{x}) = \underline{\hspace{2cm}}$$

- d) (5) Find the mean of $\hat{\lambda}_n(\mathbf{x})$, given λ . Is $\hat{\lambda}_n$ unbiased? Y N

$$m(\lambda) = E[\hat{\lambda}_n(\mathbf{x}) \mid \lambda] = \underline{\hspace{2cm}}$$

Problem 6 (cont):

Recall $\{X_i\}_{i \leq n} \stackrel{\text{iid}}{\sim} \text{Ex}(\lambda)$.

- e) (5) Find the Fisher Information $I_n(\lambda)$ for a sample of size n
- f) (10) Evaluate the Information Inequality lower bound and the squared-error risk for $\hat{\lambda}_n(\mathbf{x})$. Does $\hat{\lambda}_n(\mathbf{x})$ attain the lower bound? Y N

Done! Have a great holiday.

$$\Phi(x) = \int_{-\infty}^x \frac{1}{\sqrt{2\pi}} e^{-z^2/2} dz:$$

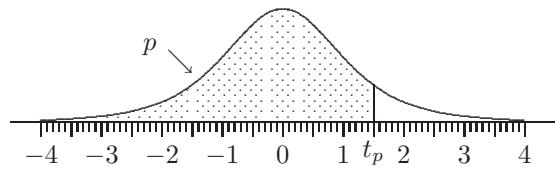
Area $\Phi(x)$ under the Standard Normal Curve to the left of x .

x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$$\begin{aligned}\Phi(0.6745) &= 0.75 & \Phi(1.6449) &= 0.95 & \Phi(2.3263) &= 0.99 & \Phi(3.0902) &= 0.999 \\ \Phi(1.2816) &= 0.90 & \Phi(1.9600) &= 0.975 & \Phi(2.5758) &= 0.995 & \Phi(3.2905) &= 0.9995\end{aligned}$$

Critical Values for Student's t

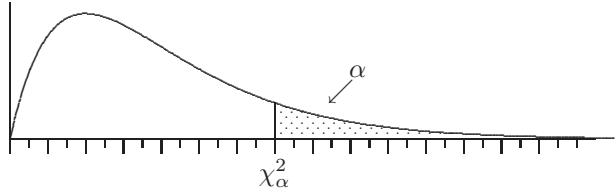
$$p = \int_{-\infty}^{t_p} c \frac{dt}{(1 + t^2/\nu)^{(\nu+1)/2}}$$



ν	$t_{.60}$	$t_{.70}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$	$t_{.9999}$
1	0.325	0.727	1.376	1.9626	3.078	6.314	12.76	31.82	63.66	318.3	636.6	3183.
2	0.289	0.617	1.061	1.3862	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.277	0.584	0.978	1.2498	1.638	2.353	3.182	4.541	5.841	10.22	12.92	22.20
4	0.271	0.569	0.941	1.1896	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.267	0.559	0.920	1.1558	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.265	0.553	0.906	1.1342	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.263	0.549	0.896	1.1192	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.262	0.546	0.889	1.1081	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.261	0.543	0.883	1.0997	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.260	0.542	0.879	1.0931	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.260	0.540	0.876	1.0877	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.259	0.539	0.873	1.0832	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.259	0.538	0.870	1.0795	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.258	0.537	0.868	1.0763	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.258	0.536	0.866	1.0735	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.258	0.535	0.865	1.0711	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.257	0.534	0.863	1.0690	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.257	0.534	0.862	1.0672	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.257	0.533	0.861	1.0655	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.257	0.533	0.860	1.0640	1.325	1.725	2.086	2.528	2.845	3.552	3.85	4.539
21	0.257	0.532	0.859	1.0627	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.256	0.532	0.858	1.0614	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.256	0.532	0.858	1.0603	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.256	0.531	0.857	1.0593	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.256	0.531	0.856	1.0584	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.256	0.531	0.856	1.0575	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.256	0.531	0.855	1.0567	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.256	0.530	0.855	1.0560	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.256	0.530	0.854	1.0553	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.256	0.530	0.854	1.0547	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.255	0.529	0.851	1.0500	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
60	0.254	0.527	0.848	1.0455	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
120	0.254	0.526	0.845	1.0409	1.289	1.658	1.980	2.358	2.617	3.160	3.373	3.837
∞	0.253	0.524	0.842	1.0364	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719

Critical Values for χ^2

$$\alpha = \int_{\chi_\alpha^2}^{\infty} c x^{\nu/2-1} e^{-x/2} dx$$



ν	$\chi^2_{.50}$	$\chi^2_{.25}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$	$\chi^2_{.001}$	$\chi^2_{.0005}$	$\chi^2_{.0001}$
1	0.4549	1.8233	2.7055	3.8415	5.0239	6.6349	7.87940	10.8276	12.1157	15.1367
2	1.3863	2.7726	4.6052	5.9915	7.3778	9.2103	10.5966	13.8155	15.2018	18.4207
3	2.3660	4.1083	6.2514	7.8147	9.3484	11.3449	12.8382	16.2662	17.7300	21.1075
4	3.3567	5.3853	7.7794	9.4877	11.1433	13.2767	14.8603	18.4668	19.9974	23.5127
5	4.3515	6.6257	9.2364	11.0705	12.8325	15.0863	16.7496	20.5150	22.1053	25.7448
6	5.3481	7.8408	10.6446	12.5916	14.4494	16.8119	18.5476	22.4577	24.1028	27.8563
7	6.3458	9.0371	12.0170	14.0671	16.0128	18.4753	20.2777	24.3219	26.0178	29.8775
8	7.3441	10.219	13.3616	15.5073	17.5345	20.0902	21.9550	26.1245	27.8680	31.8276
9	8.3428	11.389	14.6837	16.9190	19.0228	21.6660	23.5894	27.8772	29.6658	33.7199
10	9.3418	12.549	15.9872	18.3070	20.4831	23.2092	25.1882	29.5883	31.4198	35.5640
11	10.341	13.701	17.2750	19.6751	21.9200	24.7249	26.7568	31.2641	33.1366	37.3670
12	11.340	14.845	18.5493	21.0260	23.3366	26.2169	28.2995	32.9095	34.8213	39.1344
13	12.340	15.984	19.8119	22.3620	24.7356	27.6882	29.8195	34.5282	36.4778	40.8707
14	13.339	17.117	21.0641	23.6848	26.1189	29.1412	31.3193	36.1233	38.1094	42.5793
15	14.339	18.245	22.3071	24.9958	27.4884	30.5779	32.8013	37.6973	39.7188	44.2632
16	15.338	19.369	23.5418	26.2962	28.8453	31.9999	34.2672	39.2524	41.3081	45.9249
17	16.338	20.489	24.7690	27.5871	30.1910	33.4087	35.7185	40.7902	42.8792	47.5664
18	17.338	21.605	25.9894	28.8693	31.5264	34.8053	37.1565	42.3124	44.4338	49.1894
19	18.338	22.718	27.2036	30.1435	32.8523	36.1909	38.5823	43.8202	45.9731	50.7955
20	19.337	23.828	28.4120	31.4104	34.1696	37.5662	39.9968	45.3147	47.4985	52.3860
21	20.337	24.935	29.6151	32.6706	35.4789	38.9322	41.4011	46.7970	49.0108	53.9620
22	21.337	26.039	30.8133	33.9244	36.7807	40.2894	42.7957	48.2679	50.5111	55.5246
23	22.337	27.141	32.0069	35.1725	38.0756	41.6384	44.1813	49.7282	52.0002	57.0746
24	23.337	28.241	33.1962	36.4150	39.3641	42.9798	45.5585	51.1786	53.4788	58.6130
25	24.337	29.339	34.3816	37.6525	40.6465	44.3141	46.9279	52.6197	54.9475	60.1403
26	25.336	30.435	35.5632	38.8851	41.9232	45.6417	48.2899	54.0520	56.4069	61.6573
27	26.336	31.528	36.7412	40.1133	43.1945	46.9629	49.6449	55.4760	57.8576	63.1645
28	27.336	32.620	37.9159	41.3371	44.4608	48.2782	50.9934	56.8923	59.3000	64.6624
29	28.336	33.711	39.0875	42.5570	45.7223	49.5879	52.3356	58.3012	60.7346	66.1517
30	29.336	34.800	40.2560	43.7730	46.9792	50.8922	53.6720	59.7031	62.1619	67.6326
40	39.336	45.616	51.8051	55.7585	59.3417	63.6907	66.7660	73.4020	76.0946	82.0623
50	49.335	56.334	63.1671	67.5048	71.4202	76.1539	79.4900	86.6608	89.5605	95.9687
60	59.335	66.981	74.3970	79.0819	83.2977	88.3794	91.9517	99.6072	102.695	109.503
70	69.335	77.577	85.5270	90.5312	95.0232	100.425	104.215	112.317	115.578	122.755
80	79.334	88.130	96.5782	101.879	106.629	112.329	116.321	124.839	128.261	135.782
90	89.334	98.650	107.565	113.145	118.136	124.116	128.299	137.208	140.782	148.627
100	99.334	109.14	118.498	124.342	129.561	135.807	140.169	149.449	153.167	161.319

Name	Notation	pdf/pmf	Range	Mean μ	Variance σ^2
Beta	$\text{Be}(\alpha, \beta)$	$f(x) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1} (1-x)^{\beta-1}$	$x \in (0, 1)$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$
Binomial	$\text{Bi}(n, p)$	$f(x) = \binom{n}{x} p^x q^{n-x}$	$x \in 0, \dots, n$	np	npq $(q = 1 - p)$
Exponential	$\text{Ex}(\lambda)$	$f(x) = \lambda e^{-\lambda x}$	$x \in \mathbb{R}_+$	$1/\lambda$	$1/\lambda^2$
Gamma	$\text{Ga}(\alpha, \lambda)$	$f(x) = \frac{\lambda^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\lambda x}$	$x \in \mathbb{R}_+$	α/λ	α/λ^2
Geometric	$\text{Ge}(p)$	$f(x) = p q^x$ $f(y) = p q^{y-1}$	$x \in \mathbb{Z}_+$ $y \in \{1, \dots\}$	q/p $1/p$	q/p^2 q/p^2 $(y = x + 1)$
HyperGeo.	$\text{HG}(n, A, B)$	$f(x) = \frac{\binom{A}{x} \binom{B}{n-x}}{\binom{A+B}{n}}$	$x \in 0, \dots, n$	$n P$	$n P (1-P)^{\frac{N-n}{N-1}}$ $(P = \frac{A}{A+B})$
Logistic	$\text{Lo}(\mu, \beta)$	$f(x) = \frac{e^{-(x-\mu)/\beta}}{\beta[1+e^{-(x-\mu)/\beta}]^2}$	$x \in \mathbb{R}$	μ	$\pi^2 \beta^2 / 3$
Log Normal	$\text{LN}(\mu, \sigma^2)$	$f(x) = \frac{1}{x\sqrt{2\pi\sigma^2}} e^{-(\log x - \mu)^2 / 2\sigma^2}$	$x \in \mathbb{R}_+$	$e^{\mu + \sigma^2/2}$	$e^{2\mu + \sigma^2} (e^{\sigma^2} - 1)$
Neg. Binom.	$\text{NB}(\alpha, p)$	$f(x) = \binom{x+\alpha-1}{x} p^\alpha q^x$ $f(y) = \binom{y-1}{y-\alpha} p^\alpha q^{y-\alpha}$	$x \in \mathbb{Z}_+$ $y \in \{\alpha, \dots\}$	$\alpha q/p$ α/p	$\alpha q/p^2$ $\alpha q/p^2$ $(y = x + \alpha)$
Normal	$\text{No}(\mu, \sigma^2)$	$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} e^{-(x-\mu)^2 / 2\sigma^2}$	$x \in \mathbb{R}$	μ	σ^2
Pareto	$\text{Pa}(\alpha, \epsilon)$	$f(x) = \alpha \epsilon^\alpha / x^{\alpha+1}$	$x \in (\epsilon, \infty)$	$\frac{\epsilon^\alpha}{\alpha-1}$	$\frac{\epsilon^2 \alpha}{(\alpha-1)^2(\alpha-2)}$
Poisson	$\text{Po}(\lambda)$	$f(x) = \frac{\lambda^x}{x!} e^{-\lambda}$	$x \in \mathbb{Z}_+$	λ	λ
Snedecor F	$F(\nu_1, \nu_2)$	$f(x) = \frac{\Gamma(\frac{\nu_1+\nu_2}{2})(\nu_1/\nu_2)^{\nu_1/2}}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})} \times$ $x^{\frac{\nu_1-2}{2}} \left[1 + \frac{\nu_1}{\nu_2} x\right]^{-\frac{\nu_1+\nu_2}{2}}$	$x \in \mathbb{R}_+$	$\frac{\nu_2}{\nu_2-2}$	$\left(\frac{\nu_2}{\nu_2-2}\right)^2 \frac{2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}$
Student t	t_ν	$f(x) = \frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}} [1 + x^2/\nu]^{-(\nu+1)/2}$	$x \in \mathbb{R}$	0	$\nu/(\nu-2)$
Uniform	$\text{Un}(a, b)$	$f(x) = \frac{1}{b-a}$	$x \in (a, b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$
Weibull	$\text{We}(\alpha, \beta)$	$f(x) = \alpha\beta x^{\alpha-1} e^{-\beta x^\alpha}$	$x \in \mathbb{R}_+$	$\frac{\Gamma(1+\alpha^{-1})}{\beta^{1/\alpha}}$	$\frac{\Gamma(1+2/\alpha)-\Gamma^2(1+1/\alpha)}{\beta^{2/\alpha}}$

Extra worksheet, if needed: