1. Suppose the set $B$ is a subset of set $A$. Show that the complement of $A$ is a subset of the complement of $B$, in other words $A^C \subset B^C$.

2. Suppose we have 2 sets of cards, each numbered 1 to 10, with one being red and one being blue, and they are combined into one set of 20 cards. Select one card at random. Let $A$ be the event that a card with an even number is selected, $B$ the event that a blue card is selected and $C$ the event that a card with number less than 5 is selected. Describe the sample space $S$ and describe each of the following events in words:
   
   (a) $A \cap B \cap C$
   (b) $A \cap (B \cup C)$
   (c) $B \cap C^C$
   (d) $A \cup B \cup C$
   (e) $A^C \cap B^C \cap C^C$

3. Suppose a number $x$ will be drawn from the whole real line, $(-\infty, \infty)$, and let $A$, $B$ and $C$ be the events represented by the following subsets of $S$:

   (a) $A = \{x : 1 \leq x \leq 5\}$
   (b) $B = \{x : 3 < x \leq 7\}$
   (c) $C = \{x : x \leq 0\}$

   Describe each of the following events as a set of real numbers:

   (a) $A^C$
   (b) $A \cup B$
   (c) $B \cap C^C$
   (d) $A^C \cap B^C \cap C^C$
   (e) $(A \cup B) \cap C$

4. Three six-sided dice are rolled. The six sides of each die are numbered 1-6. Let $A$ be the event that the first die shows an even number, let $B$ be the event that the second die shows an even number, and let $C$ be the event that the third die shows an even number. Also, for each $i = 1, \ldots, 6$, let $A_i$ denote the event that the first die shows number $i$, and similarly $B_i$ for the second die and $C_i$ for the third. Express each of the following events in terms of the named events described above:
(a) The event that all three dice show even numbers
(b) The event that no die shows an even number
(c) The event that at least one die shows an odd number
(d) The event that at most two dice show odd numbers
(e) The event that the sum of all three dice is no greater than 5

5. Consider two events A and B such that \( P(A) = \frac{1}{3}, P(B) = \frac{1}{2} \). Determine the value of \( P(B \cap A^C) \) for each of the following conditions: (a) A and B are disjoint, (b) \( A \subset B \) and (c) \( P(A \cap B) = \frac{1}{8} \).

6. If the probability that student A will fail a certain statistics examination is 0.5, the probability that student B will fail the examination is 0.2, and the probability that both student A and student B will fail the examination is 0.1, what is the probability that at least one of these students will fail the examination?

7. Consider two events A and B with \( P(A) = 0.4 \) and \( P(B) = 0.7 \). Determine the maximum and minimum possible values of \( P(A \cap B) \) and the conditions under which each of these values is attained.

8. Prove that for every 2 events A and B, the probability that exactly one of the two events will occur is given by the expression

\[
P(A) + P(B) - 2P(A \cap B).
\]

9. Prove that for every two events A and B,

\[
P(A) = P(A \cap B) + P(A \cap B^C).
\]

10. A point \((x, y)\) is to be selected from the square S containing all points \((x, y)\) such that \(0 \leq x \leq 1\) and \(0 \leq y \leq 1\). Suppose that the probability that the selected point will belong to any specified subset of S is equal to the area of that subset. Find the probability of each of the following subsets: (a) the subset of points such that \((x - 0.5)^2 + (y - 0.5)^2 \geq 0.25\); (b) the subset of points such that \(0.5 < x + y < 1.5\); (c) the subset of points such that \(y \leq 1 - x^2\) (d) the subset of points such that \(x = y\).

11. If two balanced dice are rolled, what is the probability that the sum of the two numbers that appear will be odd?
12. If three fair coins are tossed, what is the probability that all three faces will be the same?

13. If four dice are rolled, what is the probability that each of the four numbers that appear will be different?

14. If six dice are rolled, what is the probability that each of the six different numbers will appear exactly once?

15. A box contains 24 light bulbs, of which 2 are defective. If a person selects 10 bulbs at random, without replacement, what is the probability that both defective bulbs will be selected?

16. A deck of 52 cards contains 4 aces. If the cards are shuffled and distributed in a random manner to four players so that each player receives 13 cards, what is the probability that all 4 aces will be received by the same player?

17. Which of the following 2 numbers is larger, \( \binom{50}{26} \) or \( \binom{50}{28} \)?

18. Which of the following 2 numbers is larger, \( \binom{50}{24} \) or \( \binom{50}{24} \)?