Probability
Math 641, Spring 2019

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Course Description: This is a graduate course in Probability from a measure-theoretic point of view and is designed to be sequel to STA 711 or Math 631. The course covers fundamental ideas about stochastic processes and their analysis. The main topics include: martingales, Markov chains from an advanced viewpoint, ergodic theory, Brownian motion and its connection to PDEs, Donsker's theorem. See the schedule below for more details.

Prerequisites: The course requires knowledge of measure theory. Either STA711, MATH631, or an equivalent course is a prerequisite. It is also assumed that students have had an introduction to probability (at an undergraduate level) so that they are familiar with common probability distributions and basic concepts. A weak background in probability could be overcome concurrently with some extra study, but see the instructor if this applies to you.

Tentative schedule of topics:

- Martingales

    - Sub- and super-martingales
    - Almost sure convergence, upcrossing inequality
    - L^p convergence, Doob's inequality
    - Uniform integrability
    - Optional stopping theorem
    - Important examples and applications
    - Burkholder-Davis-Gundy inequality

- Markov Chains

    - Markov property, strong markov property
    - Construction, examples
    - Recurrence and transience
    - Convergence to equilibrium, mixing times

- Ergodic Theory

    - Birkhoff ergodic theorem
    - Subadditive ergodic theorem
    - Law of large numbers
    - Applications

- Brownian Motion

    - Construction
    - Sample path properties, zero set

- – Strong Markov property, reflection principle
- – Donsker's theorem and applications
- – multi-dimensional Brownian motion.
- – Itô's formula and application to PDEs.

- Miscellaneous topics along the way:

  - – 0-1 laws: Kolmogorov, Hewitt-Savage, Blumenthal
  - – Martingale CLT and martingale approximation method
  - – Stable laws
  - – other special topics, if time permits