Exercise (1)
Fix a number $0 < p < 1$ and let $E_1, E_2, \ldots$ be independent events, all with the same probability $P[E_n] = p$. (These are called Bernoulli trials; examples include “success” for a succession of independent patients in a clinic undergoing a treatment with probability $p$ of success, or “heads” on successive tosses of a biased coin, etc.). Give the probability mass function (at every point) and mean value of each of the following random variables; if the distribution has a name (like Binomial or Poisson) give that, too. You don’t have to DERIVE the mean if you can find the name of a distribution and if the book gives the mean etc. for that distribution.

a) $X = \{\text{Total number of } n \in 1...10 \text{ such that } E_n \text{ occurs}\}$ (number of successes in $n$ trials)

$$P[X = x] = \quad E[X] = \underline{\text{___________}}$$
Name (if any):

b) $Y = \{\text{Smallest integer } n \geq 1 \text{ such that } \cup_{i=1}^{n} E_i \text{ occurs}\}$ (trial number of first success)

$$P[Y = y] = \quad E[Y] = \underline{\text{___________}}$$
Name (if any):

c) Let $I_n = 1$ if $E_n$ occurs, otherwise $I_n = 0$ (this is called an indicator random variable), and set $Z = I_1 - I_2$:

$$P[Z = z] = \quad E[Z] = \underline{\text{___________}}$$
Name (if any):

Exercise (2)
If $X$ has a geometric distribution with mean $E[X] = 3$, find $P[X = 3]$.

Exercise (3)
Ross, Chapter 4, problem 13 (p. 174).

Exercise (4)
Ross, Chapter 4, problem 21 (p. 176).
Exercise (5)
Ross, Chapter 4, exercise 8 (p. 185).

Exercise (6)
A new treatment has an adverse reaction for a small fraction $p$ of the population, for some number $0 < p < 1$. The treatment is given to $N$ subjects in a clinical trial. Let $Y$ be the number of subjects with an adverse reaction.

a) Is the expected value of $Y$, $E[Y]$, bigger or smaller or the same as the probability of at least one adverse reaction, $P[Y > 0]$? Why?

b) Does $Y$ have EXACTLY a binomial distribution, a Poisson distribution, or something else? Why?