Exercise (1)

The proportion of the sample (5 out of the 25 trout you later caught) ought to be approximate the proportion of the population (20 out of $N$).
Some of you made more nice discussions in the homework. That’s good.

Exercise (2)

All the fish, caught or not at the first time, are equally likely to be caught in the second time. This assumption is suspicious. So we would expect the fish caught the first time are more likely to be caught at the second time. Then $N$ could be a bit more than 100.

Exercise (3)

(a) $S = (1, g), (0, g), (1, f), (0, f), (1, s), (0, s)$
(b) $A = (1, s), (0, s)$
(c) $B = (0, g), (0, f), (0, s)$
(d) $(1, s), (0, s), (1, g), (1, f)$

Exercise (4)

(a) $1 - P(S \cup F \cup G) = 1 - (28 + 26 + 16 - 12 - 4 - 6 + 2)/100 = 1/2$
(b) draw the Venn diagram to obtain the answer $= \frac{14 + 10 + 8}{100}$
(c) the probability that neither one is taking is $\frac{\binom{50}{2}}{\binom{100}{2}} = 49/198$. So the probability that at least one is taking a course is $1 - 49/198 = 149/198$.

Exercise (5)

The probability is $\frac{2! \cdot \binom{1}{4} \cdot \binom{16}{1}}{\binom{52}{2}}$

Exercise (6)

2 of the 4 are from the 5 tagged, the other 2 from the 15 untagged. Under the assumptions similar to exercise (1) and (2), The probability is $\frac{\binom{5}{2} \binom{15}{2}}{\binom{20}{4}}$

Exercise (7)

\[
\begin{align*}
F_1 & = E_1 \\
F_2 & = E_2 \cap E_1^c \\
F_3 & = E_3 \cap (E_2 \cup E_1)^c = E_3 \cap E_1^c \cap E_2^c \\
\vdots \\
F_i & = E_{i-1} \cap \bigcap_{j=1}^{i} E_j^c
\end{align*}
\]
Exercise (8)

- If we obtain a tail at the first toss, the next could be either head or tail, the number of
  the outcomes for the next \( n - 1 \) tossing is \( f_{n-1} \).
- If we first get a head, then the next must be a tail. After that it could be either head or
  tail. So the number of the outcomes for the remaining \( n - 2 \) tossing is \( f_{n-2} \).

So we have \( f_n = f_{n-1} + f_{n-2} \).

- \( P_0 = \frac{f_{10}}{2^{10}} \).

Exercise (9)

- Suppose \( S = s_1, s_2, s_3, \ldots \) all the points equally likely, then if the probability is positive,
  \( P(S) = \infty \times P(s_1) = \infty \neq 1 \). Else the probability is zero, thus \( P(S) = 0 \)

- Suppose \( P(s_i) = \left(\frac{1}{2}\right)^i \). We can easily check that this is a probability over \( S \), and all points
  have positive probability.