Exercise (1)

It is actually Example 2h on Page 314 of the textbook.

Exercise (2)

\[ X = X_1 + X_2 + \cdots X_{40} \]

where

\[ X_i = \begin{cases} 1, & \text{if } i\text{th guy has no ticket} \\ 0, & \text{otherwise} \end{cases} \]

\[ P(X_i = 1) = \frac{39^{90}}{40^{90}} \]

Hence

\[ E[X] = \frac{39^{80}}{40^{90}} \]

Exercise (3)

\[ X = X_1 + X_2 + \cdots X_{365} \]

where

\[ X_i = \begin{cases} 1, & \text{if } i\text{th day of the year is the birthday of two or more students} \\ 0, & \text{otherwise} \end{cases} \]

\[ P(X_i = 1) = 1 - P(X_i = 0) = 1 - \left( \frac{365^{25}}{365^{25}} + 25 \frac{364^{24}}{365^{25}} \right) = 0.00216 \]

Hence

\[ E[X] = 365(0.00216) = 0.7881583 \]

Exercise (4)

\[ \int_0^\infty I(x)dx = \int_0^X 1dx + \int_X^\infty 0dx = X \]

\[ E \int_0^\infty I(x)dx = \int_0^\infty EI(x)dx = \int_0^\infty P(X > x)dx = \int_0^\infty (1 - F(x))dx \]

Exercise (5)

\[ (-\log \bar{F}(t))' = -\left(\frac{\bar{F}(t)}{F(t)}\right)' = \frac{f(t)}{F(t)} = \lambda(t) \]

Integrate over \((0, t)\) on both sides, then

\[ \log \bar{F}(t) = -\int_0^t \lambda(s)ds \]

Hence

\[ 1 - F(t) = \bar{F}(t) = \exp(-\int_0^t \lambda(s)ds) \]

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Exercise (6)

\[
F_{X+Y}(a) = P(X+Y \leq a) = \int \int_{X+Y \leq a} f_X(x)f_Y(y) dx \, dy
\]

\[
= \int_0^a \int_0^{a-y} e^{-x}e^{-y} dx \, dy
\]

\[
= \int_0^a (1 - e^{-(a-y)})e^{-y} dy
\]

\[
= 1 - e^{-a} - ae^{-a}, \quad 0 < a < \infty
\]

We can see this is actually gamma(2,1)