OLS estimates for slope and intercept: \( b = \frac{\sum(X - \bar{X})(Y - \bar{Y})}{\sum(X - \bar{X})^2} \), \( a = \bar{Y} - b\bar{X} \). Also, \( SE_b = \frac{s}{\sqrt{\sum(X - \bar{X})^2}} \).

95% confidence interval for the mean (of \( Y_0 \)) at level \( X_0 \):

\[
(a + bX_0) \pm t_{n-2}^{0.025} \sqrt{\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum(X - \bar{X})^2}}
\]

95% prediction/confidence interval for an individual \( Y_0 \) at level \( X_0 \):

\[
(a + bX_0) \pm t_{n-2}^{0.025} \sqrt{\frac{1}{n} + \frac{(X_0 - \bar{X})^2}{\sum(X - \bar{X})^2} + 1}
\]

1. Auditors often are required to compare the audited (or current) value \( Y \) of an inventory item with the book (or listed) value \( X \). If a company is keeping its inventory and books up to date, there should be a strong linear relationship between these values. A sample of 10 inventory items from a certain company gave the following data. (We are interested in the model \( Y = \alpha + \beta X + e \).)

\[
\begin{align*}
\bar{Y} &= 72.1 \\
\bar{X} &= 72 \\
\Sigma(X - \bar{X})(Y - \bar{Y}) &= 54243 \\
\Sigma(X - \bar{X})^2 &= 54714 \\
s^2 &= \frac{1}{n-2}\Sigma(Y - \bar{Y})^2 = 7.10568
\end{align*}
\]

a. (2 points) Estimate the expected change in audited value for a 1-unit change in book value.

\[
b = \frac{\Sigma(X - \bar{X})(Y - \bar{Y})}{\Sigma(X - \bar{X})^2} = \frac{54243}{54714} \approx 0.9914
\]

b. (3 points) If the book value is 100, what is your best estimate for the audited value?

\[
a = \bar{Y} - b\bar{X} = 72.1 - \frac{54243}{54714}(72) \approx 0.7198
\]

If you use the rounded estimate for \( b \) from part (a) above, you will get \( a \approx 0.7192 \). This is fine, but just be aware that where you round can influence your answer slightly.

So, your best estimate is \( a + b(100) \approx 0.7198 + (0.9914)(100) \approx 99.86 \).

c. (2 points) Find a 90% confidence interval for \( \beta \).

\[
b \pm t_{0.05}^{0.025} \sqrt{\frac{s^2}{\sum(X - \bar{X})^2}}
\]

\[
0.9914 \pm 1.86 \sqrt{7.10568/54714} = (0.9702, 1.0126)
\]
d. (3 points) Give a 90% interval for the average audit value if the book value is 100.

\[
(a + bX_0) \pm t_{0.025}^{n-2} \sqrt{\frac{1}{n} + \frac{(X_0 - X)^2}{\Sigma(X - \bar{X})^2}}
\]

\[
99.859 \pm 1.86\sqrt{7.10568} \sqrt{\frac{1}{10} + \frac{(100 - 72)^2}{54714}}
\]

\[
99.859 \pm 1.6765
\]

(98.183, 101.54)