1. (4 points) As items come to the end of a production line, an inspector chooses which items are to go through a complete inspection. Ten percent of all items produced are defective. Sixty percent of all defective items go through a complete inspection, and 20% of all good items go through a complete inspection. Given that an item is completely inspected, what is the probability it is defective?

\[ D: \text{item is defective}\]
\[ I: \text{item gets complete inspection}\]

\[
P(D) = 0.10 \\
P(I|D) = 0.60 \\
P(I|\bar{D}) = 0.20
\]

\[
P(D|I) = \frac{P(I|D)P(D)}{P(I)} = \frac{P(I|D)P(D)}{P(I|D)P(D) + P(I|\bar{D})P(\bar{D})} = \frac{(0.60)(0.10)}{(0.60)(0.10) + (0.20)(0.90)} = \frac{0.06}{0.06 + 0.18} = 0.25
\]
2. A large group of people is to be checked for two common symptoms of a certain disease. It is thought that 20% of the people possess symptom $A$ alone, 30% possess symptom $B$ alone, 10% possess both symptoms, and the remainder have neither symptom. For one person chosen at random from this group, find these probabilities:

a. (2 points) that the person has neither symptom

\[
P(A \cap \bar{B}) = 0.20 \\
P(\bar{A} \cap B) = 0.30 \\
P(A \cap B) = 0.10
\]

\[
P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B) \\
P(\bar{A} \cap B) = 1 - [P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(A \cap B)] \\
= 1 - (0.20 + 0.30 + 0.10) \\
= 1 - 0.60 \\
= 0.40
\]

b. (2 points) that the person has at least one symptom

\[
P(A \cup B) = P(A \cap \bar{B}) + P(\bar{A} \cap B) + P(A \cap B) \\
= 0.20 + 0.30 + 10 \\
= 0.60
\]

or

\[
P(A \cup B) = 1 - P(\bar{A} \cap \bar{B}) \\
= 1 - 0.40 \\
= 0.60
\]

c. (2 points) that the person has both symptoms, given that he has symptom $B$

\[
P(A | B) = \frac{P(A \cap B)}{P(B)} \\
= \frac{P(A \cap B)}{P(A \cap B) + P(\bar{A} \cap B)} \\
= \frac{0.10}{0.10 + 0.30} \\
= \frac{0.10}{0.40} \\
= 0.25
\]