Problem 1

(a) The normalized likelihood function is $1320p^7(1-p)^3$.

(b) The computer code is in lab 2 material.

(c) The distribution is Beta$(8, 4)$.

(d) The mean of Beta(α, β) is $\frac{\alpha}{\alpha+\beta}$, the standard deviation is $\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$. Here the mean is $\frac{2}{3}$, standard deviation is $\sqrt{\frac{2}{117}} = 0.13$

(e) In S-plus,

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1-pbeta(0.5, 8, 4)
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would yield the result 0.89.

The answers are $0.89, 0.019, 1.6E-14$, respectively. It means that it's very likely that the clinical trial is somewhat successful(success probability greater than 0.50), while unlikely to be very successful (probability greater than 0.90). It's almost impossible that the clinical trial is not successful at all.

Page 141, Problem 8

Suppose that Y is a random variable with two possible values: $Y = 1$ if instrument 1 is used, and $Y = 2$ if instrument 2 is used. So $Pr(Y = 1) = Pr(Y = 2) = 1/2$. Then the $h_1(x)$ and $h_2(x)$ are just $g_1(x|y = 1)$ and $g_1(x|y = 2)$, respectively.

(a) The joint distribution $f(x, y)$ can be obtained by

$$\begin{aligned} f(x,y) &= g_1(x|y)f_2(y) \\ &= \{ \begin{array}{ll} x, & y=1 \\ \frac{3x^2}{2}, & y=2 \end{array} \end{aligned}$$

To determine the marginal $f_1(x)$, we usually integrate out the joint $f(x, y)$ on y:

$$f_1(x) = \int_{-\infty}^{\infty} f(x,y)dy$$

In this discrete condition we just sum it up on y. The final result is

$$f_1(x) = \{ \begin{array}{cc} x + 1.5x^2 & 0 < x < 1 \\ 0 & o.w \end{array}$$

(b)

$$\begin{aligned} g_2(y=1|x) &= \frac{f(x,y=1)}{f(x)} \\ &= \frac{2}{2+3x} \\ g_2(y=1|x=\frac{1}{4}) &= \frac{8}{11} \end{aligned}$$

Page 141, Problem 9

$$\begin{aligned}
g_2(y|x) &= \{ \begin{array}{cc} x, & y=H \\ 1-x, & y=T \end{array} \\
f(x,y) &= f_1(x)g_2(y|x) \\
&= \{ \begin{array}{cc} 6x^2(1-x), & y=H \\ 6x(1-x)^2, & y=T \end{array} \\
f_2(y=H) &= \int f(x,y=H)dx = \frac{1}{2} \\
f(x|y=H) &= \frac{f_1(x)g_2(y|x)}{\frac{1}{2}} \\
&= 12x^2(1-x)
\end{aligned}$$

Page 149, Problem 1

(a) The integration $\int f(x_1, x_2, x_3)$ should be 1, so $c = \frac{1}{3}$

(b) Integrate over x_3, the marginal of X_1 and X_2 is

$$f(x_1, x_2) = \frac{x_1 + 2x_2 + \frac{3}{2}}{3}$$

(c)

$$\begin{aligned}
g(x_3|x_1, x_2) &= \frac{f(x_1, x_2, x_3)}{f(x_1, x_2)} \\
Pr(x_3 < \frac{1}{2}|x_1 = \frac{1}{4}, x_2 = \frac{3}{4}) &= \frac{5}{13}
\end{aligned}$$

Page 149, Problem 5

$$\sum_x [f(x)]^n dx$$

Page 158, Problem 7

$$g(y) = \{ \begin{array}{cc} f(y^2)|2y| = 2ye^{-y^2}, & y > 0 \\ 0, & o.w \end{array}$$

Page 169, Problem 1

$$X_1 = Y_1, X_2 = \frac{Y_2}{Y_1}, X_3 = \frac{Y_3}{Y_2}$$

$$\begin{aligned}
g(y) &= 8y_3 \left| \begin{array}{ccc} 1 & 0 & 0 \\ -\frac{y_2}{y_1^2} & \frac{1}{y_1} & 0 \\ 0 & -\frac{y_3}{y_2^2} & \frac{1}{y_2} \end{array} \right| \\
&= \frac{8y_3}{y_1 y_2}
\end{aligned}$$

Page 169, Problem 5

Denote $Z = X + Y$, $U = X - Y$, then $X = \frac{Z+U}{2}$, $Y = \frac{Z-U}{2}$.

$$\begin{aligned} f(z,u) &= 2z \begin{vmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & -\frac{1}{2} \end{vmatrix} \\ &= z \end{aligned}$$

for $z \in [0,1]$,

$$f(z) = \int_0^z f(z,u)du = z^2$$

for $z \in [1,2]$,

$$f(z) = \int_0^{2-z} f(z,u)du = z(2-z)$$