Problem 1

$$f_n(x_i|\theta) = \{ \begin{array}{cc} \frac{1}{\theta^n}, & 0 < x_i < \theta \\ 0, & o.w \end{array}$$

define a function h as follows:

$$h[max(x_1, \cdots, x_n), \theta] = \{ \begin{array}{cc} 1, & max(x_1, \cdots, x_n) \leq \theta \\ 0, & o.w \end{array}$$

$$f_n(x_i|\theta) = \frac{1}{\theta^n} h[max(x_1, \cdots, x_n), \theta]$$

The sufficient statistics for θ is $T = max(X_1, \cdots, X_n)$.

Problem 2

(a) the likelihood function is

$$L(\lambda|X) = \lambda^n \exp^{-\lambda \sum X_i}$$

(b) The MLE is $\frac{1}{\bar{X}}$

(c)

$$\sum X_i \sim Gamma(n, \lambda)$$
$$\bar{X} \sim Gamma(n, n\lambda)$$

The r.v. $Y = Gamma(n, \lambda)$ is the time needed for the nth observation to happen, if on the average there's λ observations with in unit time. The $\frac{Y}{n}$ is still the time needed for the nth observation, but the time scale changed to be n times longer. If $n = 60$, it's like to change the measure from minute to hour. So it's distributed as $Gamma(n, n\lambda)$.

(d)

$$\mu = \frac{1}{\lambda}$$
$$\sigma^2 = \frac{1}{\lambda^2}$$