P524, 2

$$
\begin{aligned}
Q = \sum \frac{(N_i - 20)^2}{20} &= 7.4 \sim \chi^2_9 \\
1 - pchisq(7.4, 9) &\approx .6
\end{aligned}
$$

P524, 3

$$
\begin{aligned}
Q = \sum \frac{(N_i - np_i)^2}{np_i} &= 3.67 \sim \chi^2_2 \\
1 - pchisq(3.67, 2) &\approx .16
\end{aligned}
$$

P524, 4

(a)

$$
\begin{aligned}
Q = \sum \frac{(N_i - np_i)^2}{np_i} &= \frac{(x_0 - np_0)^2}{np_0} + \frac{(x_1 - np_1)^2}{np_1} \\
&= \frac{n(\bar{X}_n - p_0)^2}{p_0(1 - p_0)}
\end{aligned}
$$

(b)

$$
\begin{aligned}
H_0 : N_1, N_2 &\sim MN(n, p_1, p_2) \\
N_1 &\sim Bi(n, p_1) \\
N_2 &= N - N_1 \\
p_2 &= 1 - p_1 \\
Q &= \sum \frac{(N_i - np_i)^2}{np_i} \\
&= \frac{(X - \mu)^2}{\sigma^2} \\
&\sim \chi^2_1
\end{aligned}
$$

By central limit theorem. For $k = 2$, χ^2 test is the same as using a normal approximation to the binomial, then doing a two-sided "z-test".

P524, 7

$$
\begin{aligned}
Q = \sum \frac{(N_i - np_i)^2}{np_i} &= \frac{(18 - 500(.0227))^2}{500(.0227)} + \frac{(18 - 500(.0227))^2}{500(.0227)} + \frac{(177 - 500(.2858))^2}{500(.2858)} \\
&+ \frac{(198 - 500(.383))^2}{500(.383)} + \frac{(102 - 500(.2858))^2}{500(.2858)} + \frac{(5 - 500(.0227))^2}{500(.0227)} \\
&= 27.41 \sim \chi^2_4
\end{aligned}
$$

P-value is less than .005

P531, 1

$$\begin{aligned} log(L(\theta) &= 4N_0 log(1-\theta) + N_1 log(4\theta) + 3N_1 log(1-\theta) + 2N_2 log(\theta) \\ &+ N_2 log(6) + 2N_2 log(1-\theta) + 3N_3 log(\theta) + N_3 log(4) + N_3 log(1-\theta) + 4N_4 log(\theta) \\ \hat{\theta} &= .4 \\ Q &= \sum \frac{(N_i - np_i)^2}{np_i} = 47.81 \end{aligned}$$

P531, 2

(a)

$$\begin{aligned} H_0 : p_1 &= \theta_1^2, p_2 = \theta_2^2, p_3 = (1-\theta_1-\theta_2)^2, \\ p_4 &= 2\theta_1\theta_2, p_5 = 2\theta_1(1-\theta_1-\theta_2), p_6 = 2\theta_2(1-\theta_1-\theta_2) \\ \hat{\theta}_1 &= \frac{2N_1+N_4+N_5}{2n}, \hat{\theta}_2 = \frac{2N_2+N_4+N_6}{2n} \end{aligned}$$

(b) Prof. Wolpert solved it in class.

P537, 4

$$\begin{aligned} Q = \sum \frac{(N_i - np_i)^2}{np_i} &= \frac{(82-77.439)^2}{77.439} + \frac{(89-94.62)^2}{94.62} + \frac{(54-49.8)^2}{49.8} \\ &+ \frac{(19-22.908)^2}{22.908} + \frac{(13-17.416)^2}{17.416} + \frac{(27-21.28)^2}{21.28} \\ &+ \frac{(7-11.2)^2}{11.2} + \frac{(9-5.152)^2}{5.152} \\ &= 8.6 \sim \chi_7^2 \end{aligned}$$

P-value is between (.025, .05)

P537, 5

This is explained by Simpson's paradox.

P537, 6

(a)

	OM	YM	OF	YF
I:	50%	75%	16%	50%
II:	66%	80%	28%	64%

(b)

	O	Y
I:	43%	70%
II:	40%	67%

(c)

	Total
I:	50%
II:	60%