Spotted Owl Habitat Example

Based on the univariate logistic regression model, for each additional 10\% of old forest in ring 1, what is the estimated increase in the odds that a site is a nest site?

Recall that \( \pi \) is the probability that a site is a nest site. Our univariate model is:

\[
\logit(\hat{\pi}) = \log\left(\frac{\hat{\pi}}{1-\hat{\pi}}\right) = -4.425 + 0.0617 \times \text{percentage of mature forest in ring 1}
\]  

(1)

The book gives the following formula to solve this problem:

\[
\hat{\omega}_A/\hat{\omega}_B = \exp[\hat{\beta}(A - B)]
\]  

(2)

In this formula, A and B represent settings of the explanatory variable, which is the percentage of mature forest in our case. We’re talking about a 10\% increase, so we assume that \( A > B \), and that thus we are talking about an increase from B to A. Here \( A - B = 10 \). Then using the formula,

\[
\hat{\omega}_A/\hat{\omega}_B = \exp[0.0617 \times 10] = \exp(0.617) = 1.852
\]  

(3)

Thus the odds that a site is a nest site is increased by a factor of 1.852 when we consider a 10\% increase in mature forest.

Where does this formula come from?

As we discussed in class, let’s consider a specific case, in which we are considering comparing the odds that a site is a nest site in 50\% mature forest to the odds in 40\% mature forest.

We have

\[
\logit(\hat{\pi}_{40}) = -4.425 + 0.0617 \times 40
\]  

(4)

\[
\logit(\hat{\pi}_{50}) = -4.425 + 0.0617 \times 50
\]  

(5)

We can subtract these two equations to get:

\[
\logit(\hat{\pi}_{40}) - \logit(\hat{\pi}_{50}) = 0.0617 \times -10 = -0.617
\]  

(7)

Simplifying,

\[
\logit(\hat{\pi}_{40}) - \logit(\hat{\pi}_{50}) = \log\left[\frac{\hat{\pi}_{40}}{1-\hat{\pi}_{40}}\right] = -0.617
\]  

(8)

That is,

\[
\frac{\hat{\pi}_{40}}{1-\hat{\pi}_{40}} = \exp(-0.617) = 0.54
\]  

(9)

What does this mean? This means that the odds that a site is a nest site in a 40\% mature area for ring 1 are 0.54 times the odds that a site is a nest site in a 50\% mature area for ring 1. In other words, the odds that a site is a nest site for a 50\% mature area is \( \frac{1}{0.54} = 1.852 \) times the odds for a 40\% mature area. Note that this is the same answer as above.