Homework Solutions 2

1 Exercise 2.16

a. The sample points are (HH, TH, HT, TT).

b. Assuming the coins are balanced, each sample point is equally probable. The common probability is $\frac{1}{4}$.

c. $A=$ (HT, TH) and $B=$ (HT, TH, HH).

d. Using parts b and c, we get $P(A) = 0.25 + 0.25 = 0.5$ and $P(B) = 0.75$, $P(A \cap B) = P(A) = 0.5$. $P(A \cup B) = P(B) = 0.75$ and $P(\overline{A} \cup B) = P(S) = 1$.

2 Exercise 2.17

a. Here the order is important since $(V_1, V_2)$ is different from $(V_2, V_1)$. The sample points are of the form $(V_i, V_j)$ for $i, j = 1, 2, 3$.

b. All the points have equal prob of $\frac{1}{9}$.

c. $A=$ (same vendor gets both) = $\{ (V_1, V_1), (V_2, V_2), (V_3, V_3) \}$ and $B = \text{(vendor gets at least one)} = (V_1, V_2), (V_2, V_1), (V_2, V_3), (V_3, V_2), (V_2, V_2)$ then $P(A) = 0.33$, $P(B) = \frac{5}{9}$ and $P(A \cup B) = \frac{7}{9}$ and $P(A \cap B) = P(V_2, V_2) = \frac{1}{9}$.

3 Exercise 2.18

a. Let $N_1$ and $N_2$ be empty cans and $W_1$ and $W_2$ be the cans filled with water. Then the possible pairs are $(N_i W_j)$ along with $(N_1, N_2)$ for $i, j = 1, 2$.

b. If rod worthless, then expert is guessing and probability is $\frac{1}{9}$ and probability that each experts picks the two cans containing water is $\frac{1}{5}$.

4 Exercise 2.20

a. and b. Here order is unimportant. There are six possible outcomes, each with probability $\frac{1}{6}$.

b. $P(\text{minority hired}) = 0.5$

c. $P(\text{minority hired}) = 0.5$

5 Exercise 2.25

a. Define the events $E =$ family’s income exceeds 35353 dollars and $N =$ family’s income does not exceed 35353 dollars. One of the possible sample points would be $E_1 = (EEEE)$. There are 16 of course.

b. $A = (E_1, E_2, ..., E_{11})$

$B = (E_6, E_7, ..., E_{11})$ and $C = (E_2, E_3, ..., E_5)$.

b. By the definition of median each event is equally likely and $P(E_i) = \frac{1}{16}$, then $P(A) = \frac{11}{16}$ and $P(B) = \frac{5}{16}$ and $P(C) = \frac{1}{4}$.
6 Exercise 2.31

a. Use mn rule. The first die has six possible results and the second has six so a total of 36 possible sample points.

b. Let A = observe a sum of 7 on the two dice. There are six sample points that satisfy this and thus we get
\[ P(A) = \frac{6}{36} = 6 \]

7 Exercise 2.39

a. \[
\binom{130}{2} = 8385
\]

b. \[26 \times 26 = 676\] two letter codes.
\[26 \times 26 \times 26 = 17576\] three letter codes
\[= 18252\] total major codes available.

c. \[8385 + 130 = 8515\] required

d. yes

8 Exercise 2.41

There are \[
\binom{50}{3} = 19600
\]
ways to choose the three winners. Each way is equally probable.

a. There are \[
\binom{4}{3} = 4
\]
ways for the organizers to win all the prizes. Hence the probability is \[\frac{4}{19600}\].

b. The organizers can win exactly two of the prizes if one of the other 46 people wins one prize. Using the mn rule, there are 276 ways
\[
\binom{4}{2} \times \binom{46}{1} = 276
\]
ways for this to occur. Hence the probability is \[\frac{276}{19600}\].

c. \[
\binom{4}{1} \times \binom{46}{2} = 4140
\]
. The prob is \[\frac{4140}{19600}\].

d. \[
\binom{46}{3} = 15180
\]
. The prob is \[\frac{15180}{19600}\].

9 Exercise 2.50

\[6! \left(\frac{1}{7}\right)^6 = \frac{5}{524}\].
10 Exercise 2.51

\[ 5! \left( \frac{2}{5} \right)^4 \left( \frac{3}{5} \right)^1 = \frac{3}{125}. \]

11 Exercise 2.60

Define the events \( U \) = job is unsatisfactory and \( A \) = plumber A does the job. It is given that \( P(A) = 0.4 \), \( P(U) = 0.1 \), \( P(A|U) = 0.5 \)

a. The probability of interest is \( P(U|A) = 0.125 \)

b. \( P(\overline{U} | A) = 0.875 \)

12 Exercise 2.61

a. 0.40

b. 0.37

c. 0.1

d. 0.67

e. 0.60

f. 0.33

g. 0.90

h. 0.27

i. 0.25

13 Exercise 2.62

a. since \( P(A|B) = P(A) \), thus \( P(B|A) = P(B) \) by the formula of conditional prob.

b. If \( P(B|A) = P(B) \), then similarly, we have \( P(A|B) = P(A) \).

c. Assume \( P(A \cap B) = P(A)P(B) \), then the results follows from the ones above.

14 Exercise 2.70

A = device A detects smoke

B = device B detects smoke

a. \( P(A \cup B) = 0.97 \)

b. \( P(\text{smoke undetected}) = 1 - P(A \cup B) = 0.03 \)

15 Exercise 2.84

Applying result of exercise 2.80, let \( U = A \cup B \) and \( V = C \), then we have \( P(A \cup B \cup C) \geq 1-P(\overline{A})P(\overline{B})P(\overline{C}) \)
16 Exercise 2.94

If the victim is to be saved, a proper donor must be found within eight minutes allowing 2 minutes for transfer of blood. Thus 4 people can be typed and the patient will be saved if a proper donor is found on the first, second, third, or fourth try. Note that \( P(A) = 0.4 \), where \( A \) is the event that an a type A, Rh-positive donor is found. Then \( P(\text{saving the patient}) = P(\text{A on first trial, or first A on second, or first A on third or first A on fourth}) \), thus \( P(\text{saving patient}) = 0.4 + 0.6 \times 0.4 + (0.6)^2 \times 0.4 + (0.6)^3 \times 0.4 = 0.8704 \)

17 Exercise 2.97

a. \( \frac{1}{n} \)

b. \( \left( \frac{n-1}{n} \right) \left( \frac{1}{n-1} \right) = \frac{1}{n} \) second try
\( \left( \frac{n-1}{n} \right) \left( \frac{n-2}{n-1} \right) \left( \frac{1}{n-2} \right) = \frac{1}{n} \) third try

c. \( P(\text{gain access}) = \frac{1}{n} + \frac{1}{n} + \frac{1}{n} = \frac{3}{n} \)

18 Exercise 2.104

\( C = \text{contract lung cancer and } S = \text{worked in a shipyard} \). Then \( P(S \mid C) = 0.22 \) and \( P(S \mid \bar{C}) = 0.14 \). Also \( P(C) = 0.0004 \). Using Bayes rule, we get \( P(C \mid S) = 0.0006 \)

19 Exercise 2.114

\( A = \text{woman’s name selected from list 1 and } B = \text{woman’s name selected from list 2} \). Then \( P(A) = \frac{5}{16} \), \( P(B \mid A) = \frac{5}{9} \) and \( P(B \mid \bar{A}) = \frac{5}{9} \), now by Bayes rule, we get \( P(A \mid B) = \frac{30}{37} \).

20 Exercise 2.120

Let \( Y = \text{number of positions the spinner did not land on; } Y = 2, 3 \)
\( P(Y = 2) = \frac{2}{4} \)
\( P(Y = 3) = \frac{1}{4} \).