3.2 The simple events and corresponding \( Y \) values are
\[
\begin{array}{cc}
E_i & Y \\
HH & 2 \\
HT & -1 \\
TH & -1 \\
TT & 1 \\
\end{array}
\]
Since \( P(E_i) = \frac{1}{4} \) for each \( i \), the probability distribution for \( Y \) is
\[
\begin{array}{ccc}
y & -1 & 1 & 2 \\
p(y) & \frac{1}{4} & \frac{1}{4} & \frac{1}{4} \\
\end{array}
\]

3.13 Let \( P \) be a random variable representing the company's profit. Then \( P = C - 15 \) with probability 98/100 (when \( A \) does not occur) and \( P = C - 15 - 1000 \) with probability 2/100 (when \( A \) occurs). Then \( E(P) = (C - 15) \frac{98}{100} + (C - 15 - 1000) \frac{2}{100} = 50 \). Simplifying we have \( C - 15 - 20 = 50 \). Thus \( C = 85 \).

3.16 Since the die is fair, the probability distribution for \( Y \) is
\[
p(y) = \frac{1}{6}, \quad y = 1, 2, 3, 4, 5, 6
\]
Then
\[
E(Y) = \sum yp(y) = \frac{1}{6} (1 + 2 + ... + 6) = \frac{21}{6} = 3.5
\]
\[
E(Y^2) = \sum y^2 p(y) = \frac{1}{6} (1 + 4 + 9 + ... + 36) = \frac{91}{6} = 15.1667
\]
\[
V(Y) = E(Y^2) - [E(Y)]^2 = 15.1667 - (3.5)^2 = 2.9167
\]

3.18 Consider first the probability distribution for \( y \):
\[
\begin{array}{c|c}
y & p(y) \\
0 & .81 \\
1 & .18 \\
2 & .01 \\
\end{array}
\]
Then
\[
\mu = E(Y) = \sum yp(y) = 0(.81) + 1(.18) + 2(.01) = 0.20
\]
and
\[
\sigma^2 = E(Y^2) - \mu^2 = [\sum y^2 p(y)] - \mu^2 = [0(.81) + 1(.18) + 4(.01)] - (2)^2 = .22 - .04 = .18.
\]

3.32 Let \( Y \) be the number of successful operations, with \( n = 5 \).
   a. Use Table 1 with \( p = .8 \).
      \( P(5) = P(Y \leq 5) - P(Y \leq 4) = 1 - .672 = .328 \).
   b. For \( p = .6 \), \( P(4) = P(Y \leq 4) - P(Y \leq 3) = .922 - .663 = .259 \).
   c. For \( p = .3 \), \( P(Y < 2) = P(Y \leq 1) = .528 \).

3.42 The random variable \( Y \) is binomial with \( n = 4 \), \( p = .1 \). Hence
\[
E(Y) = np = .4
\]
and
\[
E(Y^2) = V(Y) + [E(Y)]^2 = npq + n^2p^2 = 4(.1)(.9) + (.4)^2 = .52.
\]
Then \( E(C) = 3E(Y^2) + E(Y) + 2 = 3(.52) + .4 + 2 = 3.96 \).

3.44 Let \( Y \) be the number of fish that survive. \( Y \) is binomial with \( n = 20 \) and \( p = .8 \).
   a. \( P(Y = 14) = P(Y \leq 14) - P(Y \leq 13) = .196 - .087 = .109 \)
   b. \( P(Y \geq 10) = 1 - P(Y \leq 9) = 1 - .001 = .999 \)
   c. \( P(Y \leq 16) = .589 \)
   d. \( \mu = np = 20(.8) = 16; \sigma^2 = npq = 20(.8)(.2) = 3.2 \)
3.98 Let \( Y \) be the number of customers arriving. Then \( Y \) follows a Poisson distribution with \( \lambda = 7 \). We perform the calculations exactly, however one could just as easily use table 3 appendix III.

a. \[ P(Y \leq 3) = p(0) + p(1) + p(2) + p(3) = \frac{7^0}{0!} + \frac{7^1}{1!} + \frac{7^2}{2!} + \frac{7^3}{3!} = .0818. \]

b. \[ P(Y \geq 2) = 1 - P(Y \leq 1) = 1 - \frac{7^0}{0!} - \frac{7^1}{1!} = 1 - 8e^{-7} = .9927. \]

c. \[ P(Y = 5) = \frac{7^5}{5!} = .1277. \]

3.99 Let \( S = \) total service time = 10\( Y \). From 3.98 we know that \( Y \sim \text{Poisson}(7) \). Therefore,

\[
E(S) = 10E(Y) = 10(\lambda) = 10(7) = 70.
\]

\[
V(S) = (10)^2V(Y) = 100(\lambda) = 100(7) = 700.
\]

\[
P(S > 150) = P(10Y > 150) = P(Y > 15) = 1 - P(Y \leq 15) = 1 - 0.9986 = 0.002
\]

where Table 3 was used to find \( P(Y \leq 15) \). So, we infer that it is unlikely that the total service time will exceed 2.5 hours.

3.106 The binomial probabilities for \( n = 20 \) and \( p = .05 \) are obtained by using the table as in previous exercises. However, for large \( n \) and small \( p \) such that \( \lambda = np \) is less than 7, the following approximation can be used:

\[ P(Y = r) \approx \frac{e^{-\lambda} \lambda^r}{r!} \]

where \( \lambda = np \) and \( r = 0, 1, 2, \ldots, n \)

The exact binomial probabilities and their Poisson approximations are shown in the accompanying table. In this case, \( \lambda = np = 20(.05) = 1 \).

\[
\begin{array}{ccc}
\text{y} & p(y) & p(y) \\
& (\text{Exact Binomial}) & (\text{Poisson Approximation}) \\
0 & .358 & .368 \\
1 & .378 & .368 \\
2 & .189 & .184 \\
3 & .059 & .061 \\
4 & .013 & .015 \\
\end{array}
\]

Notice that the approximation is not too bad, even though \( n \) is fairly small.

4.6

a. The properties of a distribution function are satisfied since:

(1) \( F(-\infty) = 0 \).

(2) \( F(\infty) = 1 - e^{-\infty} = 1 - 0 = 1 \).

(3) \( F(y_1) - F(y_2) = e^{-y_2} - e^{-y_1} \), which is positive if \( y_1 > y_2 \).

b. By Definition 4.3,

\[
f(y) = F'(y) = \begin{cases} 2y e^{-y} & \text{for } y > 0 \\ 0 & \text{for } y \leq 0 \end{cases}
\]

c. \( P(Y \geq 2) = 1 - P(Y < 2) = 1 - P(Y \leq 2) \), since the probability at any particular point is 0. Thus,

\[
P(Y \geq 2) = 1 - F(2) = 1 - (1 - e^{-4}) = e^{-4}.
\]

d. \( P(Y > 1|Y \leq 2) = \frac{P(Y \leq 2)}{P(Y \leq 2)} \)

So we need

\[
P(Y > 1|Y \leq 2) = F(2) - F(1) = (1 - e^{-4}) - (1 - e^{-1}) = e^{-1} - e^{-4}.
\]

Next we get \( P(Y \leq 2) = 1 - e^{-4} \) (using part b.). Thus

\[
P(Y > 1|Y \leq 2) = \frac{e^{-1} - e^{-4}}{1 - e^{-4}}.
\]

4.12

a. \( F(\infty) = \int_{-\infty}^{\infty} f(y) \, dy = \int_{-1}^{0} .2 \, dy + \int_{0}^{\frac{3}{2}} (.2 + cy) \, dy = .2y |_{-1}^{0} + \left[ .2y + \frac{cy^2}{2} \right]_{0}^{\frac{3}{2}} \)

\[
= .2 + .2 + \frac{3}{2} = 1 \text{ so that } c = 1.2 \text{ and the density function is}
\]

\[
f(y) = \begin{cases} .2, & -1 < y \leq 0 \\ .2 + 1.2y, & 0 < y < 1 \\ 0, & \text{elsewhere} \end{cases}
\]
b. \( F(y) = 0 \) for \( y < -1 \).
\[
F(y) = \int_{-\infty}^{y} f(t) \, dt = \int_{-1}^{y} .2 \, dt = .2t \bigg|_{-1}^{y} = .2y + .2 \quad \text{for} \ -1 \leq y \leq 0
\]
\[
F(y) = \int_{-\infty}^{y} f(t) \, dt = \int_{-1}^{y} .2 \, dt + \int_{y}^{0} (.2 + 1.2t) \, dt = .2 + [.2t + .6t^2]_{y}^{0}
\]
\[
= .2 + .2y + .6y^2 \quad \text{for} \ 0 \leq y \leq 1. \ F(y) = 1 \quad \text{for} \ y > 1.
\]
Collecting results, we have
\[
F(y) = \begin{cases} 
0, & y < -1 \\
.2(y + 1), & -1 \leq y \leq 0 \\
.2(1 + y + 3y^2), & 0 < y \leq 1 \\
1, & y > 1
\end{cases}
\]
c. The graphs of \( f(y) \) and \( F(y) \) are shown in Figures 4.8 and 4.9.

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure4_8}
\caption{Figure 4.8}
\end{figure}

\begin{figure}[h]
\centering
\includegraphics[width=0.4\textwidth]{figure4_9}
\caption{Figure 4.9}
\end{figure}

\[ d. \quad F(-1) = .2(-1 + 1) = 0, \quad F(0) = .2(0 + 1) = .2, \quad F(1) = .2(1 + 1 + 3) = .2(5) = 1 \]
\[ e. \quad P(0 \leq Y \leq .5) = F(.5) - F(0) = .2[1 + .5 + 3(.25)] - .2 = .2(2.25) - .2 = .25 \]
\[ f. \quad P(Y > .5|Y > .1) = \frac{P(Y > .5)}{P(Y > .1)} = \frac{.45}{.24} = .714 \]

4.22 a. To solve for \( c \), we need to consider \( \int_{0}^{1} cy^2(1 - y)^4 \, dy = 1 \).

Integrating, we have \( c \left( \frac{y^5}{5} - \frac{y^4}{4} + \frac{y^3}{3} - \frac{y^2}{2} + y \right) \bigg|_{0}^{1} = 1 \).

\[ c \left[ \left( \frac{1}{5} \right) - 1 + \left( \frac{1}{3} \right) - \left( \frac{1}{2} \right) + 1 \right] = 1. \]
\[ c = (1) \left( \frac{50.0}{6} \right) = 105. \]

b. \( E(Y) = 105 \int_{0}^{1} y(1 - y)^4 \, dy = 105 \left[ \left( \frac{y^5}{5} \right) + \left( -\frac{4y^3}{3} \right) + y^2 + \left( -\frac{2y^2}{3} + \frac{y^2}{6} \right) \right]_{0}^{1} = \frac{3}{8}. \)