Solutions for quiz 6

1. (a) 

\[ V(X) = E(X^2) - (E(X))^2 = 6 - (2)^2 = 6 - 4 = 2 \]

(b) X and Y are independent. Therefore,

\[ E(XY) = E(X)E(Y) = 2 \cdot 1 = 2 \]

(c) X and Y are independent and hence \( \text{Cov}(X, Y) = 0 \). Therefore,

\[ V(2X + 3Y) = V(2X) + V(3Y) = 4V(X) + 9V(Y) \]
\[ V(Y) = E(Y^2) - (E(Y))^2 = 3 - 1 = 2 \]
\[ V(2X + 3Y) = 4 \cdot 2 + 9 \cdot 2 = 26 \]

(d) 

\[ \text{Cov}(2X + 3Y, X) = 2\text{Cov}(X, X) + 3\text{Cov}(Y, X) \]
\[ = 2V(X) = 4 \]

(e) 

\[ \rho(2X + 3Y, X) = \frac{\text{Cov}(2X + 3Y, X)}{\sqrt{V(2X + 3Y)} \sqrt{V(X)}} \]
\[ = \frac{4}{\sqrt{26} \sqrt{2}} \approx .555 \]