STA113 Homework #4
Spring 2003 Feb 20, 2003

Homework 4

Due: Friday 2:00pm, Feb 28, 2003

0. Turn in the simulation study report from Lab5.

1. 6.52

2. 7.32 (Take a look of example 7.11(c) in the textbook. Remember to use continuity correction in the approximation)

3. The joint probability distribution $p(x, y)$ is given by

$$p(-1, 0) = p(0, -1) = p(0, 1) = p(1, 0) = 1/4.$$

Show that the correlation between x and y is zero, but x and y are not independent.
Hint: To show that x and y are not independent, it is enough to show that there exists a pair (x_0, y_0) such that $p(x_0, y_0) \neq p_x(x_0)p_y(y_0)$.

4. Annie and Alvie have agreed to meet between 5pm and 6pm for dinner at Great Hall. Let x = Annie's arrival time and y = Alvie's arrival time. Suppose x and y are independent with each uniformly distributed on the interval $[5, 6]$.

(a) What is the joint pdf of x and y?

(b) What is the probability that they both arrive between 5:15 and 5:45?

(c) If the first one to arrive will wait only 10 min before leaving to eat elsewhere, what is the probability that they have dinner at Great Hall?
Hint: The event of interest is $A = \{(x, y) : |x - y| \leq 1/6\}$.

5. Two components of a minicomputer have the following joint pdf for their useful lifetime x and y:

$$f(x, y) = \begin{cases} xe^{-x(1+y)} & x \geq 0 \text{ and } y \geq 0 \\ 0 & \text{otherwise} \end{cases}$$

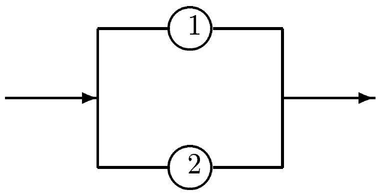
(a) What is the probability that the lifetime x of the first component exceeds 3?

(b) Are x and y independent? Explain.
Hint: (1) First calculate the marginal pdf's of random variables x and y, then check whether the joint pdf $f(x, y)$ is equal to the product of two marginals.
(2) When you calculate the marginal density of y, you will encounter a integration like this form: $\int_0^\infty x^{\alpha-1}e^{-x/\beta}dx$, which is equal to

$$\beta^\alpha\Gamma(\alpha)\int_0^\infty \frac{1}{\beta^\alpha\Gamma(\alpha)}x^{\alpha-1}e^{-x/\beta}dx = \beta^\alpha\Gamma(\alpha).$$

Here we use the fact that the function inside the integral is a Gamma(α, β) density function, therefore the integral is equal 1.
(3) $\Gamma(n + 1) = n!$, therefore, $\Gamma(2) = 1$.

6. (Let's redo the following problem from the exam.) Consider a system consisting of two components as in the figure below.



The system works if *either 1 or 2 works*. Suppose the lifetime of each component, measured in hours, is a random variable x_i, $i = 1, 2$, with density function

$$f_{x_i}(x) = \begin{cases} \frac{1}{\beta_i} e^{-x/\beta_i}, & \text{if } x \geq 0; \\ 0, & \text{if } x < 0. \end{cases}$$

where $\beta_1 = 2$ hours and $\beta_2 = 1.5$ hours and assume that the components are independent.

(a) Let $t =$ the lifetime of the system. Find the density function of the random variable t.

(b) Calculate the probability that the system will operate for more than 6 hours, using the density function calculated at (a).
Hint: Use the cumulative distribution function method. First calculate the probability of the event $\{t \leq t_0\}$ and then differentiate.