STA113 Homework #5
Spring 2003 Feb 28, 2003

Homework 5

Due: Friday 2:00pm, Mar 7, 2003

0. Turn in the lab report for Lab 6.

1. 8.8

2. 8.9

3. Let $y_1, y_2, \ldots, y_n$ be uniformly distributed on the interval 0 to θ. Recall that in class we show that the MLE of θ is $\hat{\theta} = \max(y_i)$.

(a) Let $w = max(y_i)$. Find the density function of w.
Hint: Use the fact that $w \leq w_0$ if and only if each $y_i \leq w_0$ to derive the CDF of w.

(b) Use the result in (a) to show that the MLE for θ is biased.

(c) Propose an unbiased estimator for θ.
Hint: Try the moment estimator.

4. 8.17

5. $n = 100$ random samples of water from a fresh water lake were taken and the calcium concentration (milligrams per liter) measured. A 95% CI on the mean calcium concentration is $0.49 \leq \mu \leq 0.82$.

(a) Would a 99% CI calculated from the same sample data been longer or shorter?

(b) Consider the following statement: There is a 95% chance that μ is between 0.49 and 0.82. Is this statement correct? Explain your answer.

6. The life in hours of a 75-watt light bulb is known to be normally distributed with $\sigma = 25$ hours. A random sample of 20 bulbs has a mean life of $\bar{y} = 1014$ hours.

(a) Construct a 95% CI on the mean life.

(b) Suppose we wanted the total width of the CI on mean life to be six hours at 95% confidence. What sample size should be used?