Solution for HW3

STA113, ISDS

Total 23 points

5.4

4 points

Hints:

- a. Notice $\int_{-\infty}^{\infty} f(y)dy = \int_0^{\infty} \frac{1}{c}e^{-y/2}dy + \int_{-\infty}^{0} \frac{1}{c}e^{y/2}dy = 1$.
- b.

$$F(y) = \begin{cases} 1 - \frac{1}{2}e^{-y/2} & y \geqslant 0 \\ \frac{1}{2}e^{y/2} & y < 0 \end{cases}$$

- c. From b, we have $F(y)$ and hence $F(1) = 1 - \frac{1}{2}e^{-1/2} = 0.6967347$
- d. Notice $P(Y > 0.5) = 1 - P(Y < 0.5) = 1 - F(0.5)$

5.10

3 points

Hints:

- $\mu = \int_{-\infty}^{\infty} y \cdot f(y)dy$
- $\sigma^2 = \int_{-\infty}^{\infty} (y-\mu)^2 \cdot f(y)dy$
- $P(\mu - 2\sigma < y < \mu + 2\sigma) = F(\mu + 2\sigma) - F(\mu - 2\sigma)$ And compare this to 0.95.

5.21

2 points

Hints:

- The density function for uniformly distributed random variable y over the interval $[a, b]$ is $f(y) = \frac{1}{b-a}$.
- a. $\mu = \int_{-\infty}^{\infty} y \cdot f(y)dy \quad \& \quad \sigma^2 = \int_{-\infty}^{\infty} (y-\mu)^2 \cdot f(y)dy$.
- b. $F(y) = \int_a^y \frac{1}{b-a} dx$

5.32

3 points

Hints:

- Denote X the number of push-button terminal switches demanded daily. And let $Y = \frac{X-E(X)}{sd(X)} = \frac{X-200}{50}$. Then Y is a normal distribution with mean 0 and standard deviation 1.
- Calculate $P(Y < \frac{90-200}{50}$ and $P(\frac{225-200}{50} < Y < \frac{275-200}{50})$.
- Find y such that $P(Y < y) = 0.94$, then calculate $50y + 200$.

5.43

3 points

Hints:

- Notice the cumulative distribution function for a Weibull distribution is $F(y) = 1 - e^{\frac{-y^\alpha}{\beta}}$.
- $P(y \geqslant 3) = 1 - P(y < 3)$
- $\mu = \beta^{1/\alpha}\Gamma(\frac{\alpha+1}{\alpha})$.
- $\sigma^2 = \beta^{2/\alpha}\left[\Gamma(\frac{\alpha+2}{\alpha}) - \Gamma^2(\frac{\alpha+1}{\alpha})\right]$.

- $P(\mu - 2\sigma \leqslant y \leqslant \mu + 2\sigma) = F(\mu + 2\sigma) - F(\mu - 2\sigma)$.

- Notice $\Gamma(\alpha + 1) = \alpha \cdot \Gamma(\alpha)$ and $\Gamma(0.5) = \sqrt{\pi}$.

5.57

3 points

Hints:

- $\mu = \beta^{1/\alpha}\Gamma(\frac{\alpha+1}{\alpha})$ and notice $\Gamma(1) = 1$.

- $\sigma^2 = \beta^{2/\alpha}\left[\Gamma(\frac{\alpha+2}{\alpha}) - \Gamma^2(\frac{\alpha+1}{\alpha})\right]$.

- $P(y < 6) = F(6) = 1 - e^{\frac{-6^\alpha}{\beta}}$.

- $P(y > 10) = 1 - P(y < 10)$.

5.85

3 points

Hints:

- Notice the probability density function for a beta-type random variable is $f(y) = \frac{y^{\alpha-1}(1-y)^{\beta-1}}{B(\alpha,\beta)} \quad 0 \leqslant y \leqslant 1$.

- $\mu = \frac{\alpha}{\alpha+\beta}$.

- $\sigma^2 = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$.

- $F(0.8) = \sum_{y=\alpha}^{n} p(y)$ where $p(y)$ is a binomial probability distribution with $n = \alpha + \beta - 1 = 10$ and $p = 0.8$. We are about to find $1 - F(0.8) = \sum_{y=0}^{8} p(y)$. Check Table 2 of Appendix II.

Extra

2 points

a. Let Y denote the number of failures of a testing instrument from contamination particles on the product in an 8-hour shift. Then Y has a Poisson Distribution with mean

$$E(Y) = 8 * 0.02 = 0.16$$

Hence

$$P(Y = 0) = e^{-0.16} = 0.8521438$$

b. Let Y denote the number of failures of a testing instrument from contamination particles on the product in an 24-hour shift. Then Y has a Poisson Distribution with mean

$$E(Y) = 24 * 0.02 = 0.48$$

Hence

$$P(Y \geqslant 1) = 1 - P(Y = 0) = 1 - e^{-0.48} = 0.3812166$$