1. Consider Gaussian random variable $X \sim N(\mu, I_m)$. Suppose we wish to test simultaneously the $m$ null hypotheses $H_i : \mu_i = 0$ against the two-sided alternatives $H'_i : \mu_i \neq 0$. A simple test procedure would be to reject $H_i$ if $|X_i| \geq z_{\alpha/2}$ where $z_{\alpha/2}$ is such that $\Phi(z_{\alpha/2}) = 1 - \alpha/2$ and $\Phi(\cdot)$ is the standard normal CDF. Assume that $\mu_1 = \cdots = \mu_{m_0} = 0$ and $\mu_{m_0+1} = \cdots = \mu_m = d$ where $d$ is just a fixed nonzero number.

Derive analytical formulae for the following Type I error rates:

(a) $PCER = \mathbb{E}(V)/m$.
(b) $FWER = P(V \geq 1)$.
(c) $FDR = \mathbb{E}(V/R)$ with the convention that $0/0 = 0$. [you might find the following notation is useful: $\beta = 1 - \Phi(z_{\alpha/2} - d) + \Phi(-z_{\alpha/2} - d)$.

2. Choose your values for $m, m_0$ and $d$ in Ex(1), for example, $m = 10, m_0 = 5, d = 2$. Also choose a significant level $\alpha$. Generate a $m$-vector $X$ from $N(\mu, I_m)$. Apply Benjamini and Hochberg procedure to the multiple testing. Repeat it $n$ times and report the Monte Carlo estimate for FWER, FDR and $FNR = \mathbb{E}[T/W]$.

Do the same simulation for Bonferroni procedure.

3. Consider two bags, $H$ and $K$, with two balls each. Each ball is either black or white. A white ball is added to bag $H$ and a hidden ball is transferred at random from bag $H$ to bag $K$.

(a) What is the chance of drawing a white ball from bag $K$?
(b) Then, a hidden ball is transferred from bag $K$ to bag $H$. What is the chance now of drawing a white ball from bag $H$?

4. Consider $X \sim N(\theta, 1)$ with $|\theta| \leq m$. Show that, for the quadratic loss, $\delta^m(x) = m \tanh(mx)$ is a Bayes estimator associated with the two-point prior putting each mass on $\pm m$. 