Interpretation of the slope for a log-log regression model, Case 8.1.1

Consider the regression of log of the number of species on the log of island area in Case 8.1.1 (slope is positive).

\[ \hat{\mu}\{\log(S)|\log(A)}\} = \hat{\beta}_0 + \hat{\beta}_1 \log(A) \] (1)

If A is doubled,

\[ \hat{\mu}\{\log(S)|\log(2A)}\} = \hat{\beta}_0 + \hat{\beta}_1 \log(2A) \] (2)
\[ = \hat{\beta}_0 + \hat{\beta}_1 \log(A) + \hat{\beta}_1 \log(2) \] (3)

Then doubling area increases \( \hat{\mu}\{\log(S)|\log(A)}\} \) by \( \log(2^{\hat{\beta}_1}) \).

But note that

\[ \mu\{\log(S)|\log(A)}\} = median\{\log(S)|\log(A)}\} \text{ by symmetry} \] (4)
\[ = \log (median\{S|\log(A)}\}) \text{ since log is order preserving} \] (5)

So if A is doubled, \( \log (median\{S|\log(A)}\}) \) increases by \( \log(2^{\hat{\beta}_1}) \).

Exponentiating, we see that if area is doubled, it is estimated that the median of the number of species (S) increases by a multiplicative factor of \( 2^{\hat{\beta}_1} \). Then for \( \hat{\beta}_1 = .89 \) and \( 2^{.89} = 1.189 \), it is estimated that doubling island area is estimated to increase by number of species by a factor of 1.189 or by 19%. 

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