Find the Je®reys Prior for a location-scale family, i.e. one where
\[ f(x|a, b) = b \star f((x - a) \star b) \]
for \(-\infty < a < \infty, 0 < b < \infty\) for some pdf \(f(z)\). (Hint: Write \(f(z) = e^{-\phi(z)}\) and do everything in terms of \(\phi(z) \equiv -\log f(z)\); change variables in the expectation step to \(z = b(x - a)\).)

Let \(X_j\) be a sequence of independent Po\(\lambda\) random variables.

1. Find the maximum likelihood estimator \(\hat{\lambda}_n\) on the basis of the first \(n\) observations.
2. Show that \(\hat{\lambda}_n\) is sufficient for \(\lambda\).
3. Find the Fisher information \(I(\lambda)\).
4. On the basis of the \(n = 6\) observations \(x = \{1, 0, 2, 4, 3, 0\}\), find the 10\% Likelihoodist Interval for \(\lambda\) (i.e., the set of points \(\lambda\) where the LH function attains at least 10\% of its maximum value).
5. On the basis of the same \(n = 6\) observations as before, find the equal-tail 90\% Confidence Interval for \(\lambda\), correct to four decimal places. Show the S\texttt{plus} code needed for your answer.
6. Find the Jeffrey’s prior distribution for \(\lambda\), and, on the basis of the same \(n = 6\) observations as before, find the Bayesian posterior distribution and the posterior mean \(\bar{\lambda}_n = \text{E}[\lambda | x]\).
7. Find the Bayesian equal-tail 90\% Credible Interval for \(\lambda\), using the Jeffreys prior.
8. Find the risk functions \(R(\lambda, T)\) for both \(T = \hat{\lambda}_n\) and \(T = \bar{\lambda}_n\). Which one has lower risk?