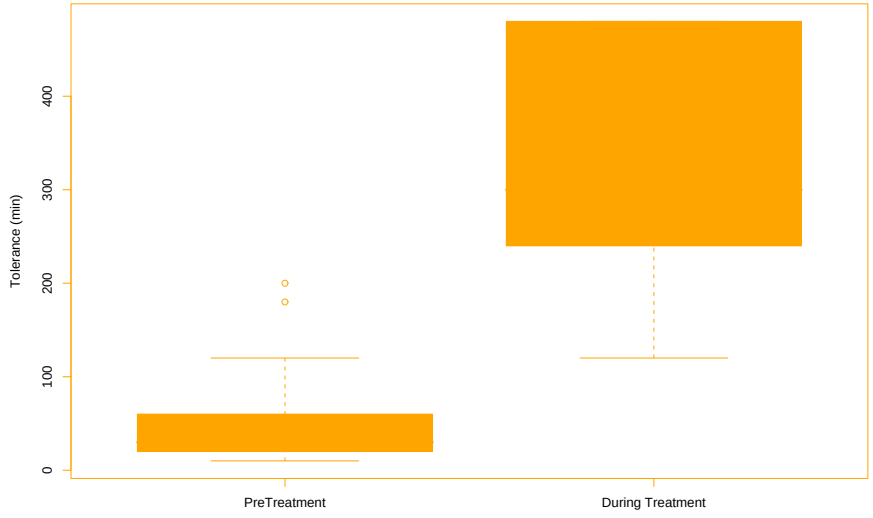
# Bayesian Inference in a Normal Population

***January 28, 2009***

Lee Chapter 2, Statistical Sleuth Chapter 4

# Example: SPF

A Sunlight Protection Factor (SPF) of 5 means an individual that can tolerate $X$ minutes of sunlight without any sunscreen can tolerate $5X$ minutes with sunscreen.



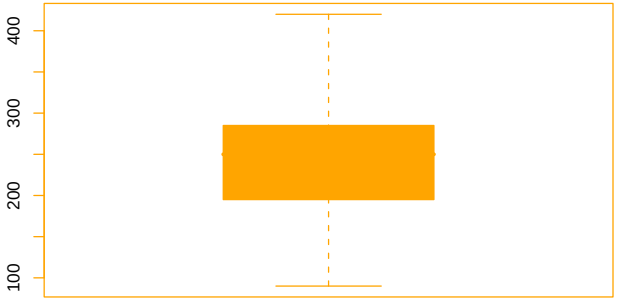
# Pairing

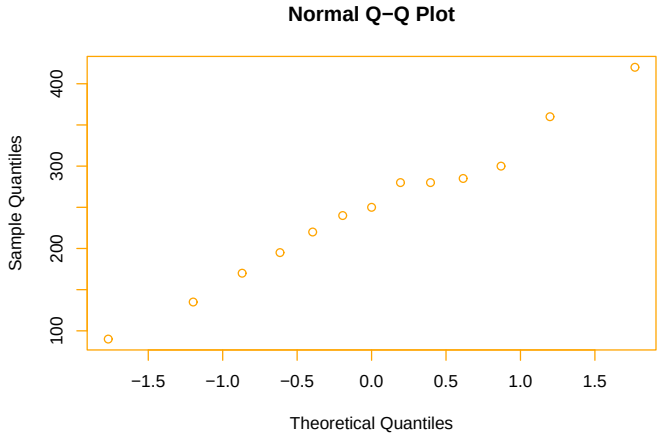
A paired design may be more powerful than two sample design because of patient to patient variability.

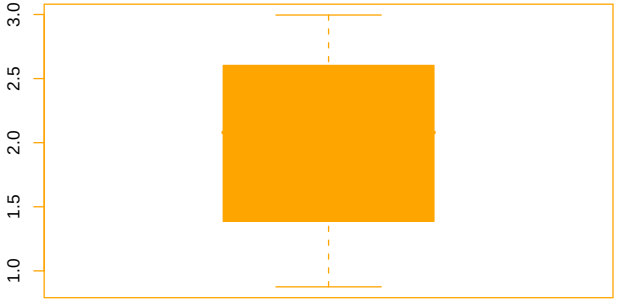
- Analysis should take into account pairing which induces dependence between observations
- use differences – or –
- use ratios or log(ratios) difference in logs

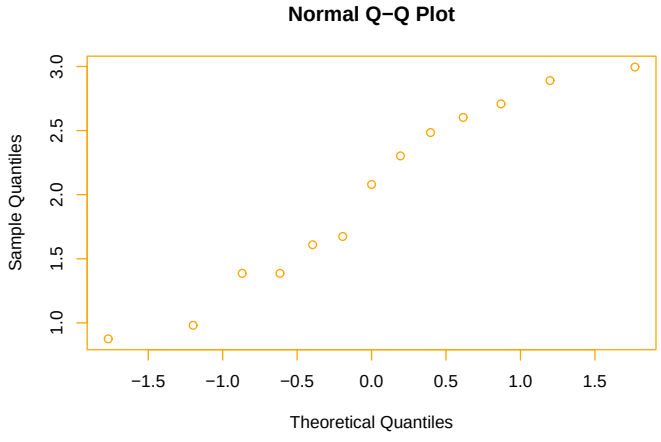
Ratios make more sense given the goals: how much longer can a person be exposed to the sun relative to their baseline $5X/X$.

# Data


100
200
300
400


Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
100
200
300
400
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5


1.0
1.5
2.0
2.5
3.0


Normal Q−Q Plot
Sample Quantiles
Theoretical Quantiles
1.0
1.5
2.0
2.5
3.0
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5

# Model for SPF

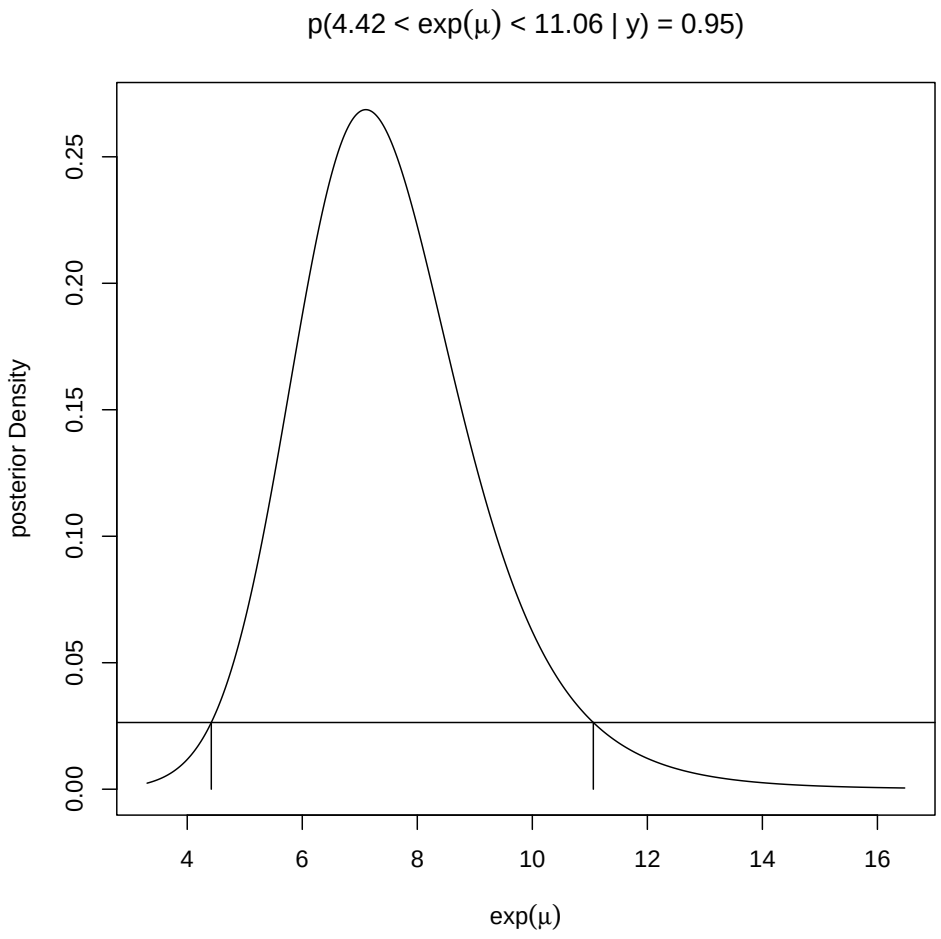
- Model `Y = log(TRT) - log(CONTROL)` as $N(\mu, 1/\lambda)$
- E(log(TRT/CONTROL)) = $\mu$ = log(SPF)
- Want distribution of $\exp \mu \equiv$SPF
- Summary statistics
  - `ybar = 1.998`
  - `s2 = 0.525`
  - `n = 13`

# Reference Posterior Distribution

Reference Prior $p(\mu, \lambda) \propto 1/\lambda)$

- $\mu \mid Y, \lambda \sim N(1.998, 1/(\lambda 13))$
- $\lambda \mid Y \sim G(12/2, 12 * .525/2)$
- $\mu \mid Y \sim t(12, 1.998, 0.525/13)$
- Distribution of $\exp(\mu)$

# Exact Posterior Distribution

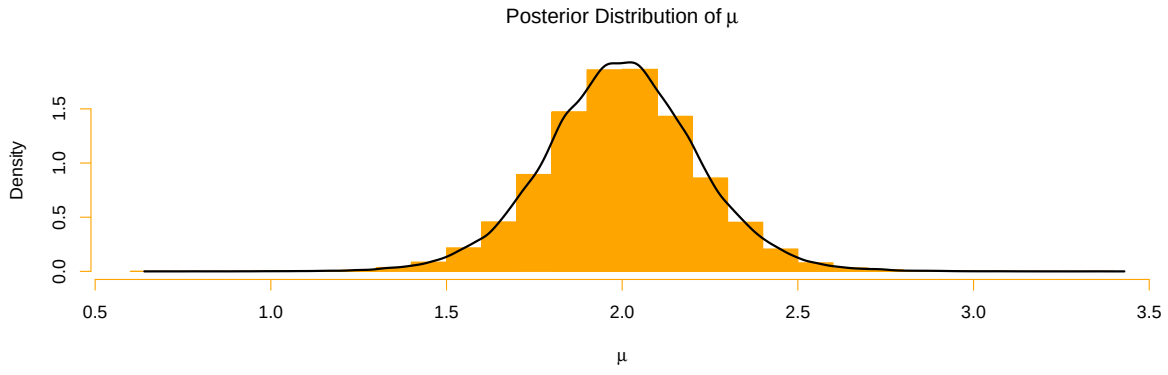


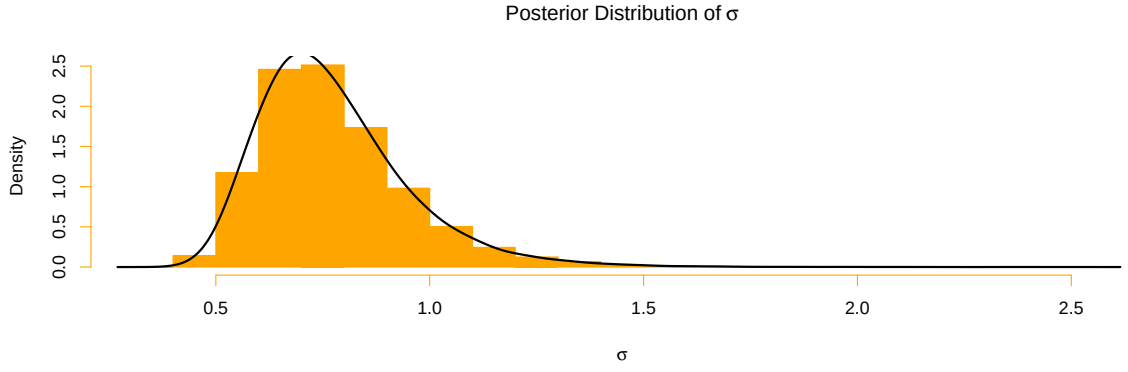
# Samples from the Posterior

To draw samples of SPF from the posterior distribution:

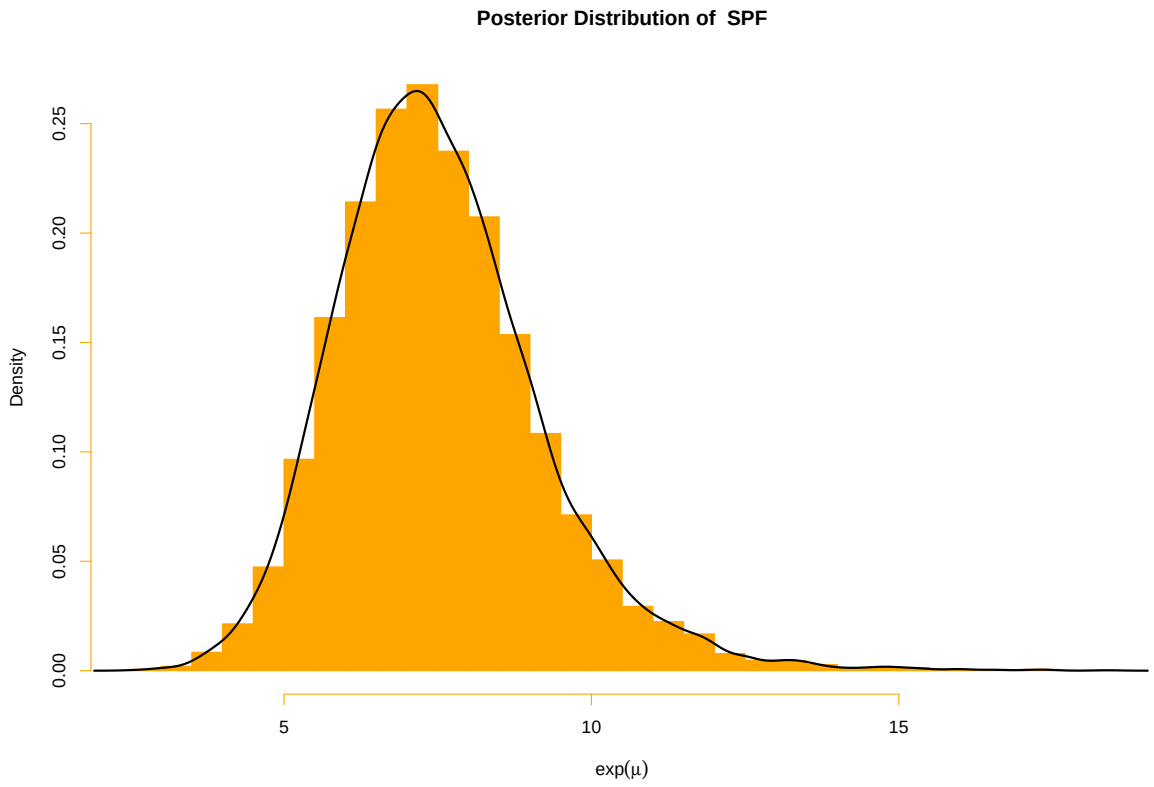
- Draw $\lambda|Y$
  ```
  lambda = rgamma(10000, (n-1)/2,
  rate=(n-1)*s2/2)
  ```
- Draw $\mu|\lambda, Y$
  ```
  mu = rnorm(10000, ybar, 1/sqrt(phi*n))
  ```
- –or– Draw $\mu|Y$ directly
  ```
  mu = ybar + rt(10000, n-1)*sqrt(s2/n)
  ```
- transform `exp(mu)`
- summarize
  ```
  quantile(exp(mu), c(.025,.5, .975))
  ```
- Use `HPDinterval` from the CODA package.

# Distributions



# Distribution for SPF




95% Probability Interval 4.76 to 11.31 (equal tail area)
95% HPD Interval 4.47 to 10.89