

STA 122 ASSIGNMENT 2

Due February 4, 2009

1. Chapter 2 of Lee, exercises 2, 3, 4, 5, 10, 16, 17, 18. For each problem where relevant, graph the posterior density in R overlayed with the prior density (if a density exists). If asked to compute HPD regions, show them on the plot. (You should be able to use the tables in Appendix B of Lee; if you have time compare them to intervals obtained using R)
2. In the production of single malt scotch whiskey, the scotch is aged for several years in oak casks. Each cask is supposed to be 40% alcohol, although there is variation from cask to cask. From the previous 5 years at a particular distillery, alcohol content of 50 casks had a mean of 39.988% and a standard deviation of 0.101 %.
 - (a) Find a conjugate Normal-Gamma prior distribution that matches the historical data. Construct a plot representing the prior distribution for the mean alcohol content.
 - (b) A sample from 10 casks from this year's production yielded a mean of 41.5% and a standard deviation of 0.105 %. Find the posterior distribution for the mean alcohol content of a cask. Graph your posterior distribution. Report a 95% HPD interval and contrast this with the HPD interval for a reference analysis.
 - (c) What is the posterior probability that the alcohol content is within 0.1% of its target of 40%?
 - (d) Suppose the distillery plans to sample a new cask. Find the posterior predictive distribution for the alcohol content of a new cask and plot the density (or estimate of it). What is the probability that its alcohol content will be within $\pm 0.1\%$ of 40.0%?
3. If ϕ has a Gamma distribution, then $1/\phi$ has an Inverse-Gamma (IG) distribution. If $\phi \sim G(v/2, S/2)$ (using the parameterization in class where the second argument of the Gamma distribution is the rate parameter) find the density for $\sigma^2 \equiv 1/\phi$. Using the parameterization in Lee what are the corresponding parameters for the IG distribution in terms of v and S ?