

STA122 Lab Session # 5

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1 Automatic HPD interval calculation using the beta-binomial example

Let $Y \sim \text{bin}(n, p)$. We assume $\text{Beta}(a, b)$ prior on p . Then the posterior distribution is given by $f(p|Y) \sim \text{Beta}(y + a, n - y + b)$. In the following example we calculate 95% credible interval of p first using a function and then automatically.

1.1 As a function

```
solve.HPD.beta = function(y, n, h, a=1, b=1, plot=T, ...){
  apost= y + a
  bpost= n - y + b
  if (apost < 1 | bpost < 1)
    warning("code assumes mode is not at 0 or 1")
  mode = (y + a - 1)/(n + a + b - 2)
  lt = uniroot(f=function(x){ dbeta(x,apost, bpost) - h},
              lower=0, upper=mode)$root
  ut = uniroot(f=function(x){ dbeta(x,apost, bpost) - h},
              lower=mode, upper=1)$root
  coverage = pbeta(ut, apost, bpost) - pbeta(lt, apost, bpost)
  if (plot) {
    th = seq(0, 1, length=500)
    plot(th, dbeta(th, apost, bpost),
         t="l", lty=1,xlab=expression(theta),
         ylab="posterior Density", ...)
    abline(h=h)
    segments(ut,0,ut,dbeta(ut,apost,bpost))
    segments(lt,0,lt,dbeta(lt,apost,bpost))
    title(bquote(paste("p(", .(round(lt, 2)), " < ", theta, " < ",
                      .(round(ut,2)), " | " , y, ") = ",
                      .(round(coverage, 2)), ")")))
  }
  return(c(lt,ut,coverage,h)) }
```

1.2 Automatic version

Here we observed 7 successes out of 10 trials. We take $Beta(1,1)$ prior on p .

```
y=7
n=10
a=1
b=1
coverage=0
plot=T
apost= y + a bpost= n - y + b
mode = (y + a - 1)/(n + a + b - 2)
h=dbeta(mode,apost,bpost)

while(coverage <=0.95){

  if (apost < 1 | bpost < 1)
    warning("code assumes mode is not at 0 or 1")

  lt = uniroot(f=function(x){ dbeta(x,apost, bpost) - h},
              lower=0, upper=mode)$root
  ut = uniroot(f=function(x){ dbeta(x,apost, bpost) - h},
              lower=mode, upper=1)$root
  coverage = pbeta(ut, apost, bpost) - pbeta(lt, apost, bpost)
  if (plot) {
    th = seq(0, 1, length=500)
    plot(th, dbeta(th, apost, bpost),
         t="l", lty=1,xlab=expression(theta),
         ylab="posterior Density")
    abline(h=h)
    segments(ut,0,ut,dbeta(ut,apost,bpost))
    segments(lt,0,lt,dbeta(lt,apost,bpost))
    title(bquote(paste("p(", .(round(lt, 2)), " < ", theta, " < ",
                      .(round(ut,2)), " | " , y, ") = ",
                      .(round(coverage, 2)), ")"))))
  }
  h=h-0.01
}
```

2 How to download and install coda package

Goto

```
packages ->install package(s)->select a mirror -> download
```

Then load the packages coda, lattice and mcmc in the R workspace using

```
> library(coda)
> library(lattice)
> library(mcmc)
```

3 Simulating from posterior distributions

To draw samples from the beta posterior and add a histogram and overlaid density function, the command is

```
> y=7
> n=10
> a=1
> b=1
> apost= y + a
> bpost= n - y + b
> x=rbeta(1000,apost,bpost)
> hist(x,freq=F)
> curve(dbeta(x,apost,bpost),add=T)
```

4 How to calculate HPDintervals using coda

```
> library(coda)
> library(lattice)
> library(mcmc)
> y=7
> n=10
> a=1
> b=1
> apost= y + a
> bpost= n - y + b
> x=rbeta(1000,apost,bpost)
> mobj<-as.mcmc(x)
> HPDinterval(mobj)
```