Sta 114 = Mth 136 : Homework 1

This is intended to be a review of the material that students in this class are expected to be familiar with, perhaps from taking a class like Duke's MTH 135 = STA 104. This material is covered in the first half of the course text, (any edition of) M.H. DeGroot and M.J. Schervish, *Probability and Statistics*.

You may discuss problems with each other but you must write up your own solutions (copying is not allowed). You won't be able to collaborate on exams, so it's very dangerous to become dependent on teamwork or on someone else's problem solving for homeworks. For full credit you must include enough work to justify your answers.

1.3 Suppose that a certain precinct contains 350 voters, of which 250 are Democrats and 100 are Republicans. If 30 voters are chosen at random from the precinct, what is the probability that exactly 18 Democrats will be selected?

1.9 Suppose that 10 cards, of which five are red and five are green, are put at random into 10 envelopes, of which seven are red and three are green, so that each envelope contains one card. Determine the probability that exactly k envelopes will contain a card with a matching color ($k = 0, 1, \dots, 10$).

1.10 Suppose that the events A and B are disjoint. Under what conditions are A^c and B^c disjoint?

2.3 Suppose that A and B are events such that $\Pr(A) = 1/3$, $\Pr(B) = 1/5$, and $\Pr(A \mid B) + \Pr(B \mid A) = 2/3$. Evaluate $\Pr(A^c \cup B^c)$.

2.13 Three students A, B, and C are enrolled in the same class. Suppose that A attends class 30% of the time, B attends class 50% of the time, and C attends class 80% of the time. If these students attend class independently of each other, what is (a) the probability that *at least* one of them will be in class on a particular day and (b) the probability that *exactly* one of them will be in class on a particular day?

2.25 Suppose that 30% of the bottles produced in a certain plant are defective. If a bottle is defective, the probability is 0.9 that an inspector will notice it and remove it from the filling line. If a bottle is not defective, the probability is 0.2 that the inspector will think that it is defective and remove it from the filling line.

(a) If a bottle is removed from the filling line, what is the probability that it is defective?

(b) If a customer buys a bottle that has not been removed from the filling line, what is the probability that it is defective?

3.15 Suppose that, on a particular day, two persons A and B arrive at a certain store independently of each other. Suppose that A remains in the store for 15 minutes and that B remains in the store for 10 minutes. If the time of arrival of each person has a uniform distribution over the hour between 9:00 A.M. and 10:00 A.M., what is the probability that A and B will be in the store at the same time?

3.21 Suppose that X and Y are iid random variables, and that each has the following pdf:

$$f(x) = \begin{cases} e^{-x} & x > 0, \\ 0 & \text{otherwise.} \end{cases}$$

Also, set $U = X/(X+Y)$ and $V = X + Y$.

(a) Determine the joint pdf of U and V.

(b) Are U and V independent?

3.22 Suppose that the random variables X and Y have the following joint pdf:

$$f(x,y) = \begin{cases} 8xy & 0 \le x \le y \le 1, \\ 0 & \text{otherwise.} \end{cases}$$

Also, set $U = X/Y$ and $V = Y$.

(a) Determine the joint pdf of U and V.

(b) Are X and Y independent?

(c) Are U and V independent?

4.7 Suppose that an automobile dealer pays an amount X (in thousands of dollars) for a used car and then sells it for an amount Y. Suppose that the random variables X and Y have the following joint pdf:

$$f(x,y) = \begin{cases} \frac{1}{36}x & 0 \le x \le y \le 6, \\ 0 & \text{otherwise.} \end{cases}$$

Determine the dealer's expected gain from the sale.

4.11 Suppose that you are going to sell cola at a football game and must decide in advance how much to order. Suppose that the demand for cola at the game, in liters, has a continuous distribution with pdf $f(x)$. Suppose that you make a profit of g cents on each liter that you sell at the game and suffer a loss of c cents on each liter that you order but do not sell. What is the optimal amount of cola for you to order so as to maximize your expected net gain?

4.27 Suppose that X is a random variable such that $\Pr[X \geq 0] = 1$ and $\mathsf{E}[X^k] < \infty$. Prove that for $k > 0$ and $t > 0$,

$$\mathsf{P}[X \geq t] \leq \frac{\mathsf{E}[X^k]}{t^k}.$$

5.13 Suppose that a pair of balanced dice are rolled 120 times; and let X denote the number of rolls on which the sum of the two numbers is 7. Use the central limit theorem to determine a value of k such that

$$\Pr[|X - 20| \leq k] \approx 0.95.$$

5.17 Suppose that X_1 and X_2 are independent random variables, and X_i has an exponential distribution with rate parameter β_i $(i = 1, 2)$, so $\Pr[X_i > x] = e^{-x\beta_i}$ for $x > 0$. Show that for each constant $k > 0$,

$$\Pr[X_1 > k\, X_2] = \frac{\beta_2}{k\beta_1 + \beta_2}.$$

5.18 Suppose that 15,000 people in a city with a population of 500,000 are watching a certain television program. If 200 people in the city are contacted at random, what is the approximate probability that four or fewer are watching the program?