Stat 376 Problem set 1-3
Due Feb 9

Pr. 1 QR Decomposition

1. Write out pseudo-code for QR decomposition.
2. Write out pseudo-code for QR decomposition with pivoting.
3. Work out the error analysis for the algorithm you wrote out to compute the QR decomposition.
4. Work out the error analysis for the algorithm you wrote out to compute the QR decomposition with pivoting.

Pr. 2 Work out an error analysis for Strassen’s algorithm and standard matrix multiplication.

Pr 3 In class we studied the following two representations for linear regression:

\[ Y_i = \beta^T X_i + \varepsilon \]
\[ Y_i = \left( \sum_{j=1}^{n} (\alpha_j x_j) \right)^T X_i + \varepsilon. \]

1. Explain the advantages and disadvantages of these two representations with respect to the dimension of the covariates and the number of observations. Solve for \( \alpha \) and \( \beta \) for both stand regression and ridge regression.
2. Redo the previous question but instead of having a linear fixed effects model you now have a linear mixed effects model.

Pr 4 Overflow

1. Explain to me why if I am working with 16 bit signed integers that

\[ 30000 + 30000 = -5536. \]

2. Explain to me the split-screen bug in Pac-Man.

Pr. 5 I give you a matrix \( Y \in \mathbb{R}^{p \times n} \) that is partially filled. I want to fill in \( Y \). This problem is sometimes called collaborative filtering (the netflix challenge) and can be solved via the following two optimization procedures

\[
\min_{k, U \in \mathbb{R}^{p \times k}, V \in \mathbb{R}^{n \times k}} \| Y - UV^T \|
\]
\[
\min_{U \in \mathbb{R}^{p \times k}, V \in \mathbb{R}^{n \times k}} \| Y - UV^T \| + \text{trace}(UV^T).
\]

Explain in words what these two optimization procedures are doing. Give me a rough back of the envelope calculation of the runtime of the algorithms that would solve these optimization problems. Is one a great deal faster than the other?