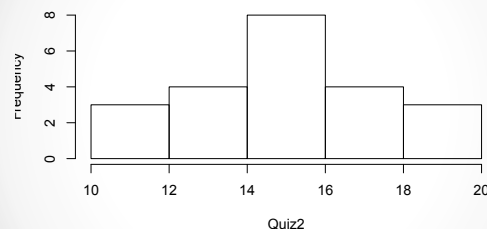


## Matching

STA 320  
Design and Analysis of Causal Studies  
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Department of Statistical Science  
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## Quiz 2

Histogram of Quiz2



```
> summary(Quiz2)
  Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
  11.0   14.0   15.0   15.5   17.0   20.0
```

## Quiz 2

- one-sided or two-sided p-value?  
(depends on question being asked)
- imputation: use observed control outcomes to impute missing treatment outcomes and vice versa.
  - class year: use observed outcomes from control sophomores to impute missing outcomes for treatment sophomores
- biased or unbiased

## Matching

- **Matching**: Find control units to “match” the units in the treatment group
- Restrict the sample to matched units
- Analyze the difference for each match (analyze as matched pair experiment)
- Useful when one group (usually the control) is much larger than the other

## Estimand

- Changes the estimand: now estimating the causal effect for **the subpopulation of treated units**
- ATE: Average treatment effect
- **ATT: Average treatment effect for the treated**
- ATC: Average treatment effect for the controls

## Exact Matching

- For **exact matching**, covariate values must match exactly
- 21 year old female in treatment group must be matched with 21 year old female in control group

## Inexact Matches

- Often, exact matching is not feasible, and matches are just as close as possible
- The farther apart the matches are, the more bias there will be
- Bias: covariate imbalance
- There are ways of adjusting for bias (ch 18)
- Can use **calipers**: only matches within a certain caliper are acceptable (remove units without an acceptable match)

## Matching

Y(1)	Y(0)	X (Age)
observed	?	19
observed	?	22
observed	?	23
?	observed	51
?	observed	30
?	observed	20
?	observed	15
?	observed	42
?	observed	21
?	observed	25

## Matching

Y(1)	Y(0)	X (Age)
observed	?	19
observed	?	22
observed	?	23
?	observed	51
?	observed	30
?	observed	20
?	observed	15
?	observed	42
?	observed	21
?	observed	25

## Matching

Y(1)	Y(0)	X (Age)
observed	?	19
observed	?	22
observed	?	23
?	observed	51
?	observed	30
?	observed	20
?	observed	15
?	observed	42
?	observed	21
?	observed	25

## Matching

Y(1)	Y(0)	X (Age)
observed	?	19
observed	?	22
observed	?	23
?	observed	51
?	observed	30
?	observed	20
?	observed	15
?	observed	42
?	observed	21
?	observed	25

## Matching

Y(1)	Y(0)	X (Age)
observed	?	19
observed	?	22
observed	?	23
?	observed	51
?	observed	30
?	observed	20
?	observed	15
?	observed	42
?	observed	21
?	observed	60

## Matching

Y(1)	Y(0)	X (Age)
observed	?	19
observed	?	22
observed	?	23
?	observed	51
?	observed	30
?	observed	20
?	observed	15
?	observed	42
?	observed	21
?	observed	60

## Matching

Y(1)	Y(0)	X (Age)
observed	?	19
observed	?	22
observed	?	23
?	observed	51
?	observed	30
?	observed	20
?	observed	15
?	observed	42
?	observed	21
?	observed	60

## Matching

Y(1)	Y(0)	X (Age)
observed	?	19
observed	?	22
observed	?	23
?	observed	51
?	observed	35
?	observed	20
?	observed	15
?	observed	42
?	observed	21
?	observed	60

uh oh...

## Ideal Matches

- Ideal: minimize total (or average) covariate distance for pairs
- Hard to do computationally, especially for large sample sizes

## "Greedy" Matching

- Greedy matching orders the treated units, and then sequentially chooses the closest control (ignoring effect on later matches)
- When doing this, helps to first match units that will be hardest to match
- One possibility: order by decreasing propensity score (treated units with highest propensity scores are most unlike controls)

## Matching with Replacement

- Matching can be done with replacement
- Pros:
  - Easier computationally (ideal matches overall same as just closest for each unit)
  - Better matches
- Cons:
  - Variance of estimator higher (controls can be used more than once, so less information)
  - Variance is harder to estimate (no longer independent)

## Matching with Replacement

- Matching with replacement is necessary if the group you want to make inferences about is the smaller group
- Matching with replacement also allows you to make inferences about the entire sample (find a match for every unit, from opposite group)
- Units more similar to those in the opposite group will be selected more

## Multiple Covariates

- With multiple covariates, how do you know which to prioritize?
- 21 year old female
- Which is a better match:
  - 18 year old female
  - 21 year old male
- Want a way to measure multivariate covariate distance

## Distance Metric

- Lots of different possible distance metrics
- Mahalanobis distance?
- Sum of squared (standardized) covariate difference in means?
- Difference in propensity scores?
- Linearized propensity score...

## Linearized Propensity Score

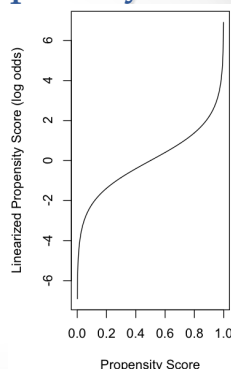
- Difference between propensity scores of 0.001 and 0.01 is larger than difference between propensity scores 0.1 and 0.109
- Better option: **linearized propensity score**, the log odds of propensity score:

$$\log\left(\frac{e(x)}{1-e(x)}\right)$$

- Logistic regression:  $\log\left(\frac{e(x)}{1-e(x)}\right) = \alpha + \beta'x$

## Linearized Propensity Score

PS	Linearized PS
0.001	-3
0.01	-2
0.10	-0.95
0.109	-0.91
0.5	0
0.9	0.95



## Linearized Propensity Score

- Note: the linearized propensity score is recommended for subclassification as well, although it isn't as important in that setting
- Won't change subclasses, but will change your view of whether a subclass is small enough

## Hybrid Matching

- In **hybrid matching**, match on more than one criteria
- Example: exact matches are required for some covariates, and other covariates are just as close as possible
  - Example: 21 year old female; look for closest age only within female controls
- Example: match on propensity score and important covariate(s)

## Multiple Matches

- Paired matching is called **1:1 matching** (1 control to 1 treated)
- If the control group is much bigger than the treatment group, can do **2:1 matching** (2 controls to 1 treatment unit), or more to one matching
- Another option: **caliper matching** in which all controls within a certain distance (based on some metric) of a treated unit are matched with that unit

## Matching

- Like propensity score estimation...
- and like subclassification....
- ... there are no "right" matches
- If the matches you choose give good covariate balance, then you did a good job!

## Decisions

- Estimating propensity score:
  - What variables to include?
  - How many units to trim, if any?
- Subclassification:
  - How many subclasses and where to break?
- Matching:
  - with or without replacement?
  - 1:1, 2:1, ... ?
  - how to weight variables / distance measure?
  - exact matching for any variable(s)?
  - calipers for which a match is "acceptable"?
  - ...

## Lalonde Data

- Analyze the causal effect of a job training program on wages
- Data on 185 treated (participated in job training program) and 2490 controls (did not participate in job training program)
- **GOAL: achieve covariate balance!**

## To Do

- Read Ch 15, 18
- Homework 4 (due Monday)