

## Assignment Mechanisms

STA 320  
Design and Analysis of Causal Studies  
Dr. Kari Lock Morgan and Dr. Fan Li  
Department of Statistical Science  
Duke University

## Announcements

- My office hours this week are CANCELLED.
- I have a bad cold and don't want to get any of you sick
- Feel free to email me with questions: [kari@stat.duke.edu](mailto:kari@stat.duke.edu)

## TA

- TA: Wenjing Shi
- Office Hours: Tuesday 5 – 7pm in Old Chem 211A
- email: [wenjing.shi@duke.edu](mailto:wenjing.shi@duke.edu)

## Review of Last Class

- SUTVA assumes no interference between units and only one version of each treatment
- $Y, W, Y_i^{\text{obs}}, Y_i^{\text{mis}}$
- The assignment mechanism (how units are assigned to treatments) is important
- Using only observed outcomes can be misleading (e.g. Perfect Doctor)
- Potential outcome framework and Rubin's Causal Model can help to clarify questions of causality (e.g. Lord's Paradox)

## Covariates

- **Covariates** are other variables that may be related to treatment assignment and/or the outcome
- Covariates are pre-treatment (measured before the treatment is applied), or permanent characteristics of a unit (gender, race)
- $K$ : number of covariates
- $\mathbf{X}$ :  $N \times K$  covariate matrix

## Uses of Covariates

- Make estimates more precise by explaining some of the variation in the outcome
- Causal effects for subgroups
- **How do they affect the assignment mechanism?**

## Covariates

- If the assignment mechanism depends on covariates...
  - treatment groups differ regarding the covariates
  - simply looking at observed averages in the treatment and control groups will be misleading
  - more advanced modeling/ techniques are needed to estimate causal effects

## Covariates

- Completely randomized experiments:
  - assignment mechanism is randomization, so does not depend on the covariates
  - estimating causal effects straightforward in this case
- Observational studies:
  - assignment mechanism usually will depend on the covariates
  - need to model dependency and take this into account

## Exercise and the Brain

- A study found that elderly people who walked at least a mile a day had significantly higher brain volume (gray matter related to reasoning) and significantly lower rates of Alzheimer's and dementia compared to those who walked less
- The article states: "Walking about a mile a day can increase the size of your gray matter, and greatly decrease the chances of developing Alzheimer's disease or dementia in older adults, a new study suggests."
- Do you trust this conclusion?
- What would help you to trust it?

Allen, N. "One way to ward off Alzheimer's: Take a Hike," msnbc.com, 10/13/10. \*

## Exercise and the Brain

- A sample of mice were divided *randomly* into two groups. One group was given access to an exercise wheel, the other group was kept sedentary
- "The brains of mice and rats that were allowed to run on wheels pulsed with vigorous, newly born neurons, and those animals then breezed through mazes and other tests of rodent IQ" compared to the sedentary mice
- Exercise increases brain activity and performance, at least in mice
- Do you trust this conclusion?

Reynolds, "Phys Ed: Your Brain on Exercise", NY Times, July 7, 2010.

## Vector Notation

- $Y_i(1)$ ,  $Y_i(0)$ ,  $W_i$  denote unit-level potential outcomes and assignment
- $\mathbf{Y}(1)$ ,  $\mathbf{Y}(0)$ ,  $\mathbf{W}$  denote the N-dimensional vectors of potential outcomes and assignments for all units

## Assignment Probabilities

- $\Pr(\mathbf{W} \mid \mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0))$ : probability of a particular *assignment* vector, given covariates and potential outcomes
- $\Pr(W_i \mid \mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0))$ : probability of unit  $i$  being assigned to the active treatment, given covariates and potential outcomes
- $p_i(\mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0)) = \Pr(W_i \mid \mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0))$

## Assignment Mechanism

Potential properties of the assignment mechanism:

- Individualistic?
- Probabilistic?
- Unconfounded?
- Known and controlled?

## Individualistic

- The probability a unit is assigned to the active treatment does not depend on the covariates or potential outcomes of the other units
- $p_i(\mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0)) = q(X_i, Y_i(1), Y_i(0))$  for some  $q$
- $\Pr(\mathbf{W} | \mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0)) \propto \prod_i q(X_i, Y_i(1), Y_i(0))^{W_i} (1 - q(X_i, Y_i(1), Y_i(0)))^{1 - W_i}$
- Not individualistic: adaptive clinical trials

## Probabilistic

- Every unit has some chance of being in either treatment group, based on covariates and potential outcomes
- $0 < p_i(\mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0)) < 1$  for all  $i$
- Non-probabilistic:
  - perfect doctor
  - lord's paradox
  - Effect of STA 101 vs STA 30 on statistical learning (treatment determined by placement exam)
- Sometimes have to eliminate units to make probabilistic

## Unconfounded

- Assignment mechanism does not depend on potential outcomes
- $\Pr(\mathbf{W} | \mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0)) = \Pr(\mathbf{W} | \mathbf{X})$
- Not unconfounded: perfect doctor
- However, if we measured a covariate (initial severity?) the doctor used to decide, could become unconfounded
- The plausibility of unconfounded often depends on the covariates collected

## Known and Controlled?

- Randomized experiments: assignment mechanism is known and controlled
- Observational studies: assignment mechanism not known or controlled

## Assignment Mechanisms

	Randomized Experiments	Classical Randomized Experiments	Observational Study	Regular Assignment Mechanism
Individualistic	?	yes	?	yes
Probabilistic	yes	yes	?	yes
Unconfounded	?	yes	?	yes
Known and controlled	yes	yes	no	

## To Do

- Read Ch 1, Ch 3
- HW 1 due Wednesday
- Quiz 1 on Wednesday