Randomized Experiments

STA 320
Design and Analysis of Causal Studies
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Review of Last Class

- Covariates (pre-treatment variables) are often important in causal inference
- Assignment probabilities:
  - $\Pr(\mathbf{W} \mid \mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0))$
  - $p_i(\mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0)) = \Pr(W_i \mid \mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0))$
- Properties of the assignment mechanism:
  - individualistic
  - probabilistic
  - unconfounded
  - known and controlled

GOAL

- For estimating causal effects, we want treatment groups that are similar regarding covariates
- Main theme of the course: create covariate balance across treatment groups
- Easiest way to accomplish this: randomized experiments

Randomized Experiment

- The assignment mechanism is random, known, and controlled by the researcher
- Because the treatments are randomly assigned, the treatment groups should all look similar regarding covariates (observed and unobserved)

Classical Randomized Experiments

- For classical randomized experiments, the assignment mechanism is individualistic, probabilistic, and unconfounded ***by design***
- For the next few weeks we'll talk only about classical randomized experiments

A Look Ahead

- Today: Designing randomized experiments
- Next week: inference from classical randomized experiments
- Following week: doing more to ensure covariate balance in experiments
- After that: observational studies
- After spring break: more complicated scenarios

Knee Surgery for Arthritis

- Researchers conducted a study on the effectiveness of a knee surgery to cure arthritis. It was randomly determined whether people got the knee surgery. Everyone who underwent the surgery reported feeling less pain.
- Is this evidence that the surgery causes a decrease in pain?

Placebo Effect

- Often, people will experience the effect they think they should be experiencing, even if they aren't actually receiving the treatment
- Example: Eurotrip
- This is known as the ***placebo effect***
- One study estimated that 75% of the effectiveness of anti-depressant medication is due to the placebo effect
- For more information on the placebo effect (it's amazing!) read The Placebo Prescription

Study on Placebos

- Blue pills are better than yellow pills
- Red pills are better than blue pills
- 2 pills are better than 1 pill
- 4 pills are better than 2 pills
- And shots are the best of all!

Placebo and Blinding

- Control groups should be given a ***placebo***, a fake treatment that resembles the active treatment as much as possible
- Using a placebo is only helpful if participants do not know whether they are getting the placebo or the active treatment
- If possible, randomized experiments should be ***double-blind***: neither the participants or the researchers involved should know which treatment the patients are actually getting

Green Tea and Prostate Cancer

- A study involved 60 men with PIN lesions, some of which turn into prostate cancer
- Half were randomized to take 600 mg of green tea extract daily, the other half were given a placebo pill
- The study was double-blind
- After one year, 1 person taking green tea and 9 taking the placebo had gotten cancer
- This is statistically significant
- Can we conclude that green tea really does help prevent prostate cancer?

Stats versus Substance

- Causal effects are statistically well-defined no matter what treatments are being compared (placebo, blinding, etc. irrelevant)
- Causal effects are substantively more relevant if experiment is well-designed with a placebo and blinding

Propensity Score

- The propensity score at x is the average unit assignment probability for units with $X_i = x$:

$$e(x) \equiv \frac{1}{N_x} \sum_{i, X_i = x} p_i(\mathbf{X}, \mathbf{Y}(1), \mathbf{Y}(0))$$

- Assuming individualistic and unconfounded assignment, the propensity score is just the probability of units with X=x getting the active treatment

Randomized Experiments

- We'll cover four types of classical randomized experiments:
  - Bernoulli randomized experiment
  - Completely randomized experiment
  - Stratified randomized experiment
  - Paired randomized experiment
- Increasingly restrictive regarding possible assignment vectors

Bernoulli

- In a Bernoulli experiment, the treatment for each unit is determined by a coin flip
- Treatment assignments for units are independent
- Usually, e(x) = ½
  - e(x) = ½ maximizes precision
  - why might e(x) differ from ½?
- e(x) can depend on covariates (rare)
- Any assignment vector, **W**, is possible

Possible Assignment Vectors

- Bernoulli: 2^N

i = 1	0	0	0	0	1	0	0	1	1	0	1	0	1	1	1	1
i = 2	0	0	0	1	0	0	1	0	0	1	1	1	0	1	1	1
i = 3	0	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1
i = 4	0	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1

- Why might this not be a good design?

Completely Randomized

- In a completely randomized experiment, sample sizes for each treatment group are fixed in advance
- N_T = size of treatment group
- N_C = size of control group
- Often $N_T = N/2$, but not always
- $e(x) = N_T/N$
- Group sizes are the only restriction

Possible Assignment Vectors

- Bernoulli: 2^N

i = 1	0	0	0	0	1	0	0	1	1	0	1	0	1	1	1	1
i = 2	0	0	0	1	0	0	1	0	0	1	1	1	0	1	1	1
i = 3	0	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1
i = 4	0	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1

- Completely randomized experiment: $\binom{N}{N_t}$

i = 1	0	0	0	0	1	0	0	1	1	0	1	0	1	1	1	1
i = 2	0	0	0	1	0	0	1	0	0	1	1	1	0	1	1	1
i = 3	0	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1
i = 4	0	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1

Stratified

- In a stratified randomized experiment, units are partitioned into blocks or strata that are similar with respect to one or more covariates
- Units are completely randomized within each block/strata
- Ensures balance for important covariate(s)
- Also called blocking
- Advice: "block what you can, randomize what you cannot"

Possible Assignment Vectors

- Bernoulli: 2^N

i = 1	0	0	0	0	1	0	0	1	1	0	1	0	1	1	1	1
i = 2	0	0	0	1	0	0	1	0	0	1	1	1	0	1	1	1
i = 3	0	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1
i = 4	0	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1

- Completely randomized experiment: $\binom{N}{N_t}$

i = 1	0	0	0	0	1	0	0	1	1	0	1	0	1	1	1	1
i = 2	0	0	0	1	0	0	1	0	0	1	1	1	0	1	1	1
i = 3	0	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1
i = 4	0	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1

- Stratified randomized experiment:

female 1	0	0	0	0	1	0	0	1	1	0	1	0	1	1	1	1
female 2	0	0	0	1	0	0	1	0	0	1	1	1	0	1	1	1
male 1	0	0	1	0	0	1	0	0	1	1	0	1	1	0	1	1
male 2	0	1	0	0	0	1	1	1	0	0	0	1	1	1	0	1

Paired

- In a paired randomized experiment, units are first matched into pairs of similar units
- Within each pair, randomize which unit is treated
- Special case of blocking
- Goal: improve covariate balance and increase precision
- Also called matched pairs experiments

Paired Experiments

- Examples:
  - same person at different points in time
  - pairs with closest values of covariates
  - twin studies

Example

- Does drinking a sports drink (e.g. Gatorade) make you run faster, as opposed to just drinking water?
- How would you design an experiment with each of the following designs?
  - Bernoulli?
  - Completely randomized?
  - Stratified?
  - Paired?

To Do

- Read Ch 4
- Bring laptops to class on Monday (and make sure you have R)
- HW 2 due next Wednesday