Sta 961: Homework #2

1. Equation (1) of the class notes *Introduction to Computer Experiments* gives explicit expressions for the Matérn correlation function $r_\nu(h \mid \theta)$ for odd half-integer $\nu = n + 1/2$ smoothness parameter, for $n = 1$ and $n = 2$, as the products of exponential function and a polynomial in $h$ of degree $n$. Do the same for $n = 3$ and $n = 4$, i.e., $\nu = 7/2$ and $\nu = 9/2$.

2. Let $c_1(x)$ and $c_2(x)$ both be even functions (i.e. satisfy $c(-x) = c(x)$) that solve the equation

$$\sum_{j=0}^{p} a_j c(j - h) = \begin{cases} 1 & h = 0 \\ 0 & h > 0 \end{cases}$$

for all integers $h \geq 0$, with $a_0 = 1$ and $a_p \neq 0$. Show that $c_1(x) = c_2(x)$ for all $x \in \mathbb{Z}$.

3. Find the function $c(x) = c(-x)$ that satisfies

$$\sum_{j=0}^{2} a_j c(j - h) = \begin{cases} 1 & h = 0 \\ 0 & h > 0 \end{cases}$$

for $a_0 = 1$, $a_1 = -5/6$, $a_2 = 1/6$.

4. Find the function $c(x) = c(-x)$ that satisfies

$$\sum_{j=0}^{2} a_j c(j - h) = \begin{cases} 1 & h = 0 \\ 0 & h > 0 \end{cases}$$

for $a_0 = 1$, $a_1 = 1/2$, $a_2 = 1/4$.

5. If the time series $X_t$ satisfies

$$X_t - (5/6)X_{t-1} + (1/6)X_{t-2} = \zeta_t$$

for $\{\zeta_t\} \overset{iid}{\sim} \text{No}(0, 1)$, and if

$$X_5 = f \quad X_6 = e \quad X_7 = d \quad X_8 = c \quad X_9 = b \quad X_{10} = a$$

for specified real numbers $a, b, c, d, e, f$, find the conditional distribution (including the values of any parameters) of $X_{12}$ given $\mathcal{F}_{10}$, i.e., given $\{X_s : s \leq 10\}$. 

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