STA 250/MTH 342 Intro to Mathematical Statistics
Lab Session 7 / March 02, 2015 / Handout

In this session we work on an example, from which it is very hard to draw samples directly.

See: `https://stat.duke.edu/courses/Spring15/sta250/labs/` for links to source code and data. Submit lab solutions via email to: `sta250@stat.duke.edu`. Any plots should be included in postscript form as attachments. The email subject must be "STA250 . . . " with ". . . " replaced by your name.

1: The problem. Now consider the following bivariate pdf.

$$f(w,v) = \text{Const} \times v^{-\frac{r+n+2}{2}} \exp\left[-\frac{rs+(n-1)s_x^2+n(\bar{x}-w)^2}{2v} - \frac{(w-a)^2}{2b^2}\right],$$

defined on $w \in (-\infty, \infty)$ and $v \in (0, \infty)$. It is fairly difficult to calculate the constant term that makes this function a pdf. But it is relatively easy to show that

- $f_1(w \mid v) = \mathsf{No}\left(\frac{nb^2\bar{x}+va}{nb^2+v}, \frac{vb^2}{nb^2+v}\right)$.
- $f_2(v \mid w) = \mathsf{InvGamma}\left(\frac{r+n}{2}, \frac{rs+(n-1)s_x^2+n(\bar{x}-w)^2}{2}\right)$.

TASK 1 Use the above two conditionals to run a Gibbs sampler and generate samples of (w, v) from $f(w, v)$. Take $a = 0, b = r = s = 1$, $n = 10$, $\bar{x} = 1.38$ and $s_x = 0.33$. Generate histograms of w samples and separately for v samples.

For you to work on the task, here are some tips.

Recall that last time we worked on Gibbs sampling and the code is here
`https://stat.duke.edu/courses/Spring15/sta250/labs/lab6/gibbs.samp.R`
you are advised to reuse some part of it.

The distribution $\mathsf{InvGamma}(\alpha, \beta)$ is referred to as the *inverse gamma distribution.* For example, it appears in question15, page 406 of the textbook. The parameters α and β should both be positive, and the pdf is

$$f_{\mathsf{InvGamma}(\alpha,\beta)}(x) = \begin{cases} \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-\alpha-1} e^{-\beta/x}, & \text{for } x > 0, \\ 0, & \text{for } x \le 0. \end{cases}$$

Recall the definition of the $\mathsf{Ga}(\alpha, \beta)$ distribution, for $\alpha, \beta > 0$. The pdf is

$$f_{\mathsf{Gamma}(\alpha,\beta)}(x) = \begin{cases} \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} e^{-\beta x}, & \text{for } x > 0, \\ 0, & \text{for } x \le 0. \end{cases}$$

One has

$$X \sim \mathsf{Ga}(\alpha, \beta) \Leftrightarrow \frac{1}{X} \sim \mathsf{InvGamma}(\alpha, \beta).$$

Therefore, the following **R** command generates a sequence x of $n = 5$ independent random variables from $\mathsf{InvGamma}(\alpha = 1.5, \beta = 2.3)$.

```
> x <- 1 / rgamma(n = 5, shape = 1.5, rate = 2.3)
> x
[1] 1.3762238 1.2532001 2.5736181 2.2477231 0.4937492
```

When you finish the task, the histograms would look like those in Figure 1.

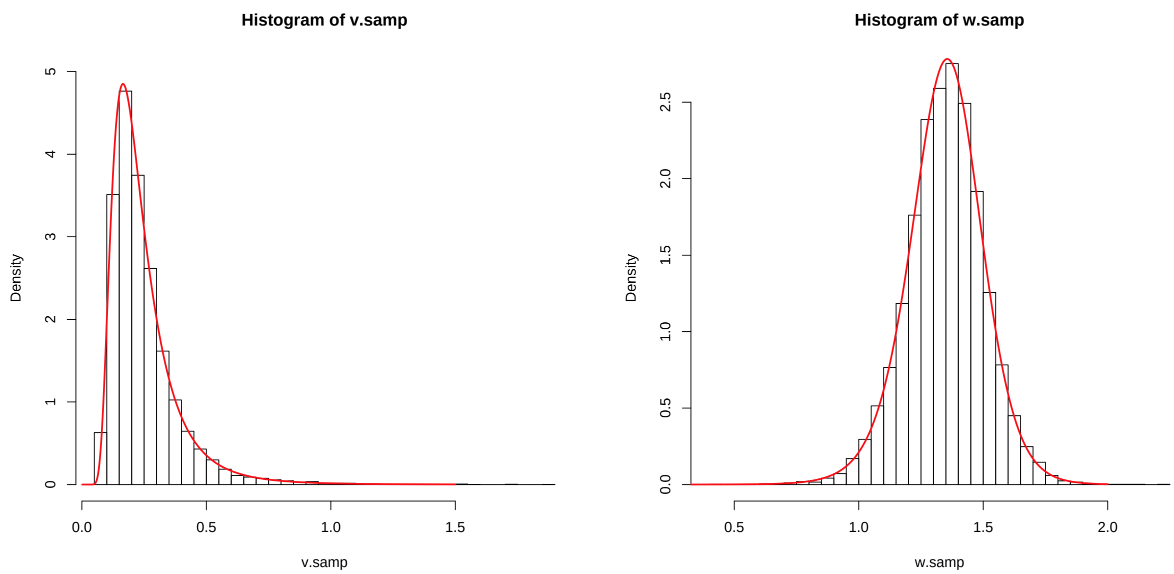

Figure 1: Model solutions. Total samples drawn: 10,000. The curves of the marginal distributions are plotted in red. You are not required to draw the red curves.

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