Final Examination[a]

STA 532: Statistical Inference

Wednesday, 2015 Apr 29, 7:00 – 10:00pm

- This is a **closed book** exam— books & phones on the floor.
- You may use a calculator and **two pages** of your own notes. Do not share calculators or notes.
- **Show** your **work**. Neatness counts. Boxing answers helps.
- **Simplify all expressions** for full credit. No unevaluated sums, integrals, maxima, or unreduced fractions.
- Dist'n & pdf/pmf tables and blank worksheet are attached.

Good luck!

Print Name Clearly: ______________________________

1.	/20	5.	/20
2.	/20	6.	/20
3.	/20	7.	/20
4.	/20	8.	/20
	/80		/80
Total:	/160		

Problem 1: The random variables $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{Po}(\theta)$ have the Poisson distribution with mean θ, for $1 \le i \le n$.

a) (2) Find the Likelihood Function. Simplify!
$L(\theta \mid \mathbf{x}) =$

b) (3) Find the Fisher Information for a sample of size n and the Reference (Jeffreys' Rule) prior density:
$I_n(\theta) =$ $\pi_J(\theta) \propto$

c) (4) Find the posterior distribution for this sample & prior in terms of the sample sum $S = \sum_{i=1}^n X_i$, by giving its name and parameter values:
$\pi_J(\theta \mid \mathbf{x}) \sim$

d) (4) Find the Variance and Bias for the posterior mean $\bar{\theta}_n^J := \mathsf{E}_J(\theta \mid \mathbf{x})$:
$\mathsf{Var}(\bar{\theta}_n^J) =$ $\beta(\theta) =$

e) (4) Find the squared-error Risk Function for this estimator:
$R(\bar{\theta}_n^J, \theta) =$

f) (3) Does $\bar{\theta}^J$ attain the Information Inequality lower bound for biased estimators? ◯ Yes ◯ No Why?

Problem 2: Let $\{X_j\} \overset{\text{iid}}{\sim} \mathsf{Ex}(\theta)$ for $1 \le j \le n$.

a) What is the likelihood function? (Simplify!)

$f(\mathbf{x}_n \mid \theta) \propto$

b) What is a one-dimensional sufficient statistic $S(\mathbf{x}_n)$?

$S(\mathbf{x}_n) =$

c) Using a Gamma prior distribution $\theta \sim \mathsf{Ga}(\alpha, \lambda)$, find the posterior distribution, by name & parameter values:

$\pi(\theta \mid \mathbf{x}_n) \sim$

d) Find the predictive pdf for one additional observation, using the same $\mathsf{Ga}(\alpha, \lambda)$ prior:

$f(x_{n+1} \mid \mathbf{x}_n) =$

Problem 3: In each part of this problem the following statistics are from a random sample of n iid random variables $\{X_i\}$ from some specified probability distribution, but the specific distribution may vary:

$$\begin{array}{lcl} \frac{1}{n}\sum X_i = A & \mid & \frac{1}{n}\sum \log X_i = C & \mid & \max X_i = E \\ \frac{1}{n}\sum (X_i - \bar{X}_n)^2 = B & \mid & \frac{1}{n}\sum \log(1-X_i) = D & \mid & \min X_i = F \end{array}$$

In each case, find a statistic T that is a function of one or more of n, A, B, C, D, E, F for which the Rejection Region of the indicated LRT or GLRT is of the form $\mathcal{R} = \{x : T(x) \geq t\}$. Express T **explicitly as a function of** n, A, B, C, D, E, F. Note T isn't unique; full credit for any function that works. **Simplify!**

a) $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{Po}(\theta)$; $H_0 : \theta = 3$ vs. $H_1 : \theta = 2$.
$T =$

b) $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{Be}(\alpha, \beta)$; $H_0 : \alpha = 1, \beta = 2$ vs. $H_1 : \alpha = 2, \beta = 1$.
$T =$

c) $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{Ga}(\alpha, 4)$; $H_0 : \alpha = 1$ vs. $H_1 : \alpha = 4$.
$T =$

d) $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{Ga}(\alpha, \lambda)$; $H_0 : \alpha = 1$ vs. $H_1 : \alpha = 4$ (λ unknown).
$T =$

e) $H_0 : \{X_i\} \overset{\text{iid}}{\sim} \mathsf{Ex}(2)$ vs. $H_1 : \{X_i\} \overset{\text{iid}}{\sim} \mathsf{Ga}(2, 1)$.
$T =$

Problem 4: Random variables $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{No}(\mu_1, \sigma^2)$ and $\{Y_i\} \overset{\text{iid}}{\sim} \mathsf{No}(\mu_2, \sigma^2)$ are normally distributed for $1 \le i \le n = 12$ with the same (unknown) variance, but the X_i's and Y_i's might not be independent of each other. Their differences $d_i = (X_i - Y_i)$ are also normally distributed and iid. Summary statistics for these $n = 12$ pairs include:

$$\begin{array}{rcl} \sum X_i = 18 & | & \sum(X_i - \bar{X}_n)^2 = 110 \\ \sum Y_i = 24 & | & \sum(Y_i - \bar{Y}_n)^2 = 88 \\ \sum d_i = -6 & | & \sum(d_i - \bar{d}_n)^2 = 154 \end{array}$$

a) Find a 90% Confidence Interval for μ_1.

b) A 90% Confidence Interval for μ_2 based on these data is $[0.5337, 3.4663]$. Explain precisely *what event* it is that has probability 90%.

c) Find a 90% Confidence Interval for $(\mu_1 - \mu_2)$, under the assumption that the X_i's and Y_i's are independent, so all the $(X_i - \mu_1)$ and $(Y_i - \mu_2)$ are iid $\mathsf{No}(0, \sigma^2)$.

Problem 4 (cont'd): Still for $1 \le i \le 12$ the X_i's, Y_i's, and $d_i = (X_i - Y_i)$ have

$$\begin{array}{rcl} \sum X_i = 18 & | & \sum (X_i - \bar{X}_n)^2 = 110 \\ \sum Y_i = 24 & | & \sum (Y_i - \bar{Y}_n)^2 = 88 \\ \sum d_i = -6 & | & \sum (d_i - \bar{d}_n)^2 = 154 \end{array}$$

d) Find a 90% Confidence Interval for $(\mu_1 - \mu_2)$, if the X_i's and Y_i's are expected to be positively correlated[1].

e) One way to test the hypothesis $H_0 : \mu_1 = \mu_2$ against $H_1 : \mu_1 \neq \mu_2$ is to Reject if zero is *not* in the confidence interval you found in parts c) or d) above. Which test would be more powerful? The one based on interval from:

○ c) ○ d) ○ Both the same power ○ It depends

Explain:

[1] For example, (X_i, Y_i) might be mileages for the ith car on two different tries; or responses of the ith subject to two different drug treatments; or sunburn scores on left and right arms of ith subject in a sunscreen test. Note there is no need to know the correlation coefficient— it should not appear in your answer.

Problem 5: Let $X \sim \mathsf{Bi}(n,p)$ be the number of successes in n independent trials, all with success probability p, and let $\phi := \log\left(\frac{p}{1-p}\right)$ be the log odds.

a) (5) Find the maximum likelihood estimator for ϕ:
$\hat{\phi}(x) =$

b) (10) For large n, $\hat{\phi}(X)$ has approximately a normal distribution. Use the delta method to find the approximate mean and variance.
$\mu_x =$ $\sigma_x^2 =$

c) (5) One way to quantify the difference between the success probabilities p and q for independent Binomial trials with $X \sim \mathsf{Bi}(n,p)$ and $Y \sim \mathsf{Bi}(m,q)$ is through the *log odds ratio*

$$\varepsilon := \log\left\{\frac{p}{1-p}\frac{1-q}{q}\right\},$$

which will be zero if $p = q$ but positive (resp, negative) if $p > q$ (resp, $p < q$). The MLE of this too will be approximately normally-distributed. Give the mean and variance:
$\mathsf{E}[\hat{\varepsilon}] \approx$ $\mathsf{Var}[\hat{\varepsilon}] \approx$

Problem 6: Sandy, a lazy student, is assigned the task of rolling a (possibly loaded) die 60 times, and reporting the outcomes. Sandy reports finding:

$1's$	$2's$	$3's$	$4's$	$5's$	$6's$
9	10	12	8	10	11

a) We wish to test $H_0 =$ [these are 60 rolls of a fair die]. Give and evaluate a test statistic $T(\mathbf{x})$ that will typically be small if H_0 is true and large under the alternative $H_1 =$ [die is unfair]:
$T(\mathbf{x}) =$

b) Give the approximate probability distribution of your test statistic $T(\mathbf{x})$, if H_0 is true, by specifying its name and parameter value(s):
$T(\mathbf{x}) \sim$

c) Give an expression (either an integral or a bit of `R` code) for the P-value for H_0. Would you reject H_0 at level $\alpha = 0.10$? Why? You should be able to answer this without a table or computer.
$P =$ Reject? ◯ Yes ◯ No

d) Sandy's instructor is suspicious that Sandy didn't actually perform the assigned task. Why?

Problem 7: $\{X_i\}_{1\le i\le n} \stackrel{\text{iid}}{\sim} \mathsf{Pa}(\theta,1)$ have the generalized Pareto distribution with shape $\theta>0$ and unit scale, with pdf and CDF for $x>0$

$$f(x \mid \theta) = \theta(1+x)^{-\theta-1} \qquad F(x \mid \theta) = 1-(1+x)^{-\theta}$$

a) Find a one-dimensional sufficient statistic for a sample of size n:
$T(\mathbf{x}) =$

b) Express the likelihood function in terms of $T(\mathbf{x})$.
$L(\theta) =$

c) Find a conjugate parametric family of prior distributions

d) Give the posterior parameters, in terms of the prior parameters, n, and the statistic T.

e) Give the posterior mean using the improper prior $\pi(\theta) = \mathbf{1}_{\{\theta>0\}}$, for a single observation of $X_1 = 1$.
$\mathsf{E}[\theta \mid X_1 = 1] =$

Problem 8: For two points each, True or False? No explanations needed.

a. T F If $X \sim \chi^2_8$ and Y are independent, and if $X + Y \sim \chi^2_{12}$, then $Y \sim \chi^2_4$.

b. T F The best test of $H_0 : \{X_i\} \overset{\text{iid}}{\sim} f_0(x)$ vs. $H_1 : \{X_i\} \overset{\text{iid}}{\sim} f_1(x)$ will reject H_0 for large values of $T(\mathbf{x}) := \sum_i \log\big(f_1(x_i)/f_0(x_i)\big)$.

c. T F If $\hat{\theta}(\mathbf{x})$ is an unbiased estimator of θ, then $e^{\hat{\theta}(\mathbf{x})}$ is an unbiased estimator of e^{θ}.

d. T F If we observe one success in two independent tries with a uniform prior for the success probability $\theta \in [0, 1]$, then the posterior probability distribution is also uniform on $[0, 1]$.

e. T F If $T(\mathbf{x})$ is a sufficient statistic, then so is $S(\mathbf{x}) := e^{T(\mathbf{x})}$.

f. T F If $\hat{\theta}(\mathbf{x})$ is the MLE of θ, then $e^{\hat{\theta}(\mathbf{x})}$ is the MLE of e^{θ}.

g. T F In "regular" statistical models the MLE $\hat{\theta}_n$ is approximately normally distributed for large n, with mean θ and variance $I_n(\theta)^{-1}$.

h. T F For small normal samples with known variance, use the t distribution to construct confidence intervals or hypothesis tests about the mean.

i. T F $[17, 42]$ is a 90% Confidence Interval for a parameter θ if and only if $\mathsf{P}[17 \le \theta \le 42] = 0.90$.

j. T F If $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{Ga}(7, \lambda)$ then $\sum X_i$ is sufficient for λ.

k. T F An estimator δ with MSE Q and bias β has variance $Q - \beta^2$.

Extra worksheet, if needed:

$$\Phi(x) = \int_{-\infty}^{x} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz:$$

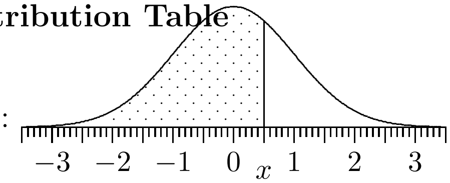


Table 5.1 Area $\Phi(x)$ under the Standard Normal Curve to the left of x.

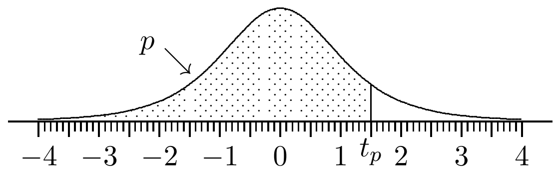
x	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

$\Phi(0.6745) = 0.75$ $\Phi(1.6449) = 0.95$ $\Phi(2.3263) = 0.99$ $\Phi(3.0902) = 0.999$

$\Phi(1.2816) = 0.90$ $\Phi(1.9600) = 0.975$ $\Phi(2.5758) = 0.995$ $\Phi(3.2905) = 0.9995$

Critical Values for Student's t

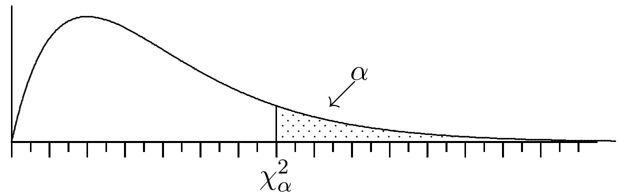
$$p = \int_{-\infty}^{t_p} c \, \frac{dt}{(1 + t^2/\nu)^{(\nu+1)/2}}$$



ν	$t_{.60}$	$t_{.70}$	$t_{.80}$	$t_{.85}$	$t_{.90}$	$t_{.95}$	$t_{.975}$	$t_{.99}$	$t_{.995}$	$t_{.999}$	$t_{.9995}$	$t_{.9999}$
1	0.325	0.727	1.376	1.9626	3.078	6.314	12.76	31.82	63.66	318.3	636.6	3183.
2	0.289	0.617	1.061	1.3862	1.886	2.920	4.303	6.965	9.925	22.33	31.60	70.70
3	0.277	0.584	0.978	1.2498	1.638	2.353	3.182	4.541	5.841	10.22	12.92	22.20
4	0.271	0.569	0.941	1.1896	1.533	2.132	2.776	3.747	4.604	7.173	8.610	13.03
5	0.267	0.559	0.920	1.1558	1.476	2.015	2.571	3.365	4.032	5.893	6.869	9.678
6	0.265	0.553	0.906	1.1342	1.440	1.943	2.447	3.143	3.707	5.208	5.959	8.025
7	0.263	0.549	0.896	1.1192	1.415	1.895	2.365	2.998	3.499	4.785	5.408	7.063
8	0.262	0.546	0.889	1.1081	1.397	1.860	2.306	2.896	3.355	4.501	5.041	6.442
9	0.261	0.543	0.883	1.0997	1.383	1.833	2.262	2.821	3.250	4.297	4.781	6.010
10	0.260	0.542	0.879	1.0931	1.372	1.812	2.228	2.764	3.169	4.144	4.587	5.694
11	0.260	0.540	0.876	1.0877	1.363	1.796	2.201	2.718	3.106	4.025	4.437	5.453
12	0.259	0.539	0.873	1.0832	1.356	1.782	2.179	2.681	3.055	3.930	4.318	5.263
13	0.259	0.538	0.870	1.0795	1.350	1.771	2.160	2.650	3.012	3.852	4.221	5.111
14	0.258	0.537	0.868	1.0763	1.345	1.761	2.145	2.624	2.977	3.787	4.140	4.985
15	0.258	0.536	0.866	1.0735	1.341	1.753	2.131	2.602	2.947	3.733	4.073	4.880
16	0.258	0.535	0.865	1.0711	1.337	1.746	2.120	2.583	2.921	3.686	4.015	4.791
17	0.257	0.534	0.863	1.0690	1.333	1.740	2.110	2.567	2.898	3.646	3.965	4.714
18	0.257	0.534	0.862	1.0672	1.330	1.734	2.101	2.552	2.878	3.610	3.922	4.648
19	0.257	0.533	0.861	1.0655	1.328	1.729	2.093	2.539	2.861	3.579	3.883	4.590
20	0.257	0.533	0.860	1.0640	1.325	1.725	2.086	2.528	2.845	3.552	3.85	4.539
21	0.257	0.532	0.859	1.0627	1.323	1.721	2.080	2.518	2.831	3.527	3.819	4.493
22	0.256	0.532	0.858	1.0614	1.321	1.717	2.074	2.508	2.819	3.505	3.792	4.452
23	0.256	0.532	0.858	1.0603	1.319	1.714	2.069	2.500	2.807	3.485	3.768	4.415
24	0.256	0.531	0.857	1.0593	1.318	1.711	2.064	2.492	2.797	3.467	3.745	4.382
25	0.256	0.531	0.856	1.0584	1.316	1.708	2.060	2.485	2.787	3.450	3.725	4.352
26	0.256	0.531	0.856	1.0575	1.315	1.706	2.056	2.479	2.779	3.435	3.707	4.324
27	0.256	0.531	0.855	1.0567	1.314	1.703	2.052	2.473	2.771	3.421	3.690	4.299
28	0.256	0.530	0.855	1.0560	1.313	1.701	2.048	2.467	2.763	3.408	3.674	4.275
29	0.256	0.530	0.854	1.0553	1.311	1.699	2.045	2.462	2.756	3.396	3.659	4.254
30	0.256	0.530	0.854	1.0547	1.310	1.697	2.042	2.457	2.750	3.385	3.646	4.234
40	0.255	0.529	0.851	1.0500	1.303	1.684	2.021	2.423	2.704	3.307	3.551	4.094
60	0.254	0.527	0.848	1.0455	1.296	1.671	2.000	2.390	2.660	3.232	3.460	3.962
120	0.254	0.526	0.845	1.0409	1.289	1.658	1.980	2.358	2.617	3.160	3.373	3.837
∞	0.253	0.524	0.842	1.0364	1.282	1.645	1.960	2.326	2.576	3.090	3.291	3.719

Critical Values for χ^2

$$\alpha = \int_{\chi^2_\alpha}^{\infty} c\, x^{\nu/2-1} e^{-x/2}\, dx$$



ν	$\chi^2_{.50}$	$\chi^2_{.25}$	$\chi^2_{.10}$	$\chi^2_{.05}$	$\chi^2_{.025}$	$\chi^2_{.01}$	$\chi^2_{.005}$	$\chi^2_{.001}$	$\chi^2_{.0005}$	$\chi^2_{.0001}$
1	0.4549	1.3233	2.7055	3.8415	5.0239	6.6349	7.87940	10.8276	12.1157	15.1367
2	1.3863	2.7726	4.6052	5.9915	7.3778	9.2103	10.5966	13.8155	15.2018	18.4207
3	2.3660	4.1083	6.2514	7.8147	9.3484	11.3449	12.8382	16.2662	17.7300	21.1075
4	3.3567	5.3853	7.7794	9.4877	11.1433	13.2767	14.8603	18.4668	19.9974	23.5127
5	4.3515	6.6257	9.2364	11.0705	12.8325	15.0863	16.7496	20.5150	22.1053	25.7448
6	5.3481	7.8408	10.6446	12.5916	14.4494	16.8119	18.5476	22.4577	24.1028	27.8563
7	6.3458	9.0371	12.0170	14.0671	16.0128	18.4753	20.2777	24.3219	26.0178	29.8775
8	7.3441	10.219	13.3616	15.5073	17.5345	20.0902	21.9550	26.1245	27.8680	31.8276
9	8.3428	11.389	14.6837	16.9190	19.0228	21.6660	23.5894	27.8772	29.6658	33.7199
10	9.3418	12.549	15.9872	18.3070	20.4831	23.2092	25.1882	29.5883	31.4198	35.5640
11	10.341	13.701	17.2750	19.6751	21.9200	24.7249	26.7568	31.2641	33.1366	37.3670
12	11.340	14.845	18.5493	21.0260	23.3366	26.2169	28.2995	32.9095	34.8213	39.1344
13	12.340	15.984	19.8119	22.3620	24.7356	27.6882	29.8195	34.5282	36.4778	40.8707
14	13.339	17.117	21.0641	23.6848	26.1189	29.1412	31.3193	36.1233	38.1094	42.5793
15	14.339	18.245	22.3071	24.9958	27.4884	30.5779	32.8013	37.6973	39.7188	44.2632
16	15.338	19.369	23.5418	26.2962	28.8453	31.9999	34.2672	39.2524	41.3081	45.9249
17	16.338	20.489	24.7690	27.5871	30.1910	33.4087	35.7185	40.7902	42.8792	47.5664
18	17.338	21.605	25.9894	28.8693	31.5264	34.8053	37.1565	42.3124	44.4338	49.1894
19	18.338	22.718	27.2036	30.1435	32.8523	36.1909	38.5823	43.8202	45.9731	50.7955
20	19.337	23.828	28.4120	31.4104	34.1696	37.5662	39.9968	45.3147	47.4985	52.3860
21	20.337	24.935	29.6151	32.6706	35.4789	38.9322	41.4011	46.7970	49.0108	53.9620
22	21.337	26.039	30.8133	33.9244	36.7807	40.2894	42.7957	48.2679	50.5111	55.5246
23	22.337	27.141	32.0069	35.1725	38.0756	41.6384	44.1813	49.7282	52.0002	57.0746
24	23.337	28.241	33.1962	36.4150	39.3641	42.9798	45.5585	51.1786	53.4788	58.6130
25	24.337	29.339	34.3816	37.6525	40.6465	44.3141	46.9279	52.6197	54.9475	60.1403
26	25.336	30.435	35.5632	38.8851	41.9232	45.6417	48.2899	54.0520	56.4069	61.6573
27	26.336	31.528	36.7412	40.1133	43.1945	46.9629	49.6449	55.4760	57.8576	63.1645
28	27.336	32.620	37.9159	41.3371	44.4608	48.2782	50.9934	56.8923	59.3000	64.6624
29	28.336	33.711	39.0875	42.5570	45.7223	49.5879	52.3356	58.3012	60.7346	66.1517
30	29.336	34.800	40.2560	43.7730	46.9792	50.8922	53.6720	59.7031	62.1619	67.6326
40	39.336	45.616	51.8051	55.7585	59.3417	63.6907	66.7660	73.4020	76.0946	82.0623
50	49.335	56.334	63.1671	67.5048	71.4202	76.1539	79.4900	86.6608	89.5605	95.9687
60	59.335	66.981	74.3970	79.0819	83.2977	88.3794	91.9517	99.6072	102.695	109.503
70	69.335	77.577	85.5270	90.5312	95.0232	100.425	104.215	112.317	115.578	122.755
80	79.334	88.130	96.5782	101.879	106.629	112.329	116.321	124.839	128.261	135.782
90	89.334	98.650	107.565	113.145	118.136	124.116	128.299	137.208	140.782	148.627
100	99.334	109.14	118.498	124.342	129.561	135.807	140.169	149.449	153.167	161.319

Name	Notation	pdf/pmf	Range	Mean μ	Variance σ^2	
Beta	$\mathsf{Be}(\alpha,\beta)$	$f(x)=\frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}x^{\alpha-1}(1-x)^{\beta-1}$	$x\in(0,1)$	$\frac{\alpha}{\alpha+\beta}$	$\frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)}$	
Binomial	$\mathsf{Bi}(n,p)$	$f(x)=\binom{n}{x}p^x q^{(n-x)}$	$x\in 0,\cdots,n$	$n\,p$	$n\,p\,q$	$(q=1-p)$
Exponential	$\mathsf{Ex}(\lambda)$	$f(x)=\lambda\,e^{-\lambda x}$	$x\in\mathbb{R}_+$	$1/\lambda$	$1/\lambda^2$	
Gamma	$\mathsf{Ga}(\alpha,\lambda)$	$f(x)=\frac{\lambda^\alpha}{\Gamma(\alpha)}x^{\alpha-1}\,e^{-\lambda x}$	$x\in\mathbb{R}_+$	α/λ	α/λ^2	
Geometric	$\mathsf{Ge}(p)$	$f(x)=p\,q^x$	$x\in\mathbb{Z}_+$	q/p	q/p^2	$(q=1-p)$
		$f(y)=p\,q^{y-1}$	$y\in\{1,...\}$	$1/p$	q/p^2	$(y=x+1)$
HyperGeo.	$\mathsf{HG}(n,A,B)$	$f(x)=\frac{\binom{A}{x}\binom{B}{n-x}}{\binom{A+B}{n}}$	$x\in 0,\cdots,n$	$n\,P$	$n\,P\,(1-P)\frac{N-n}{N-1}$	$(P=\frac{A}{A+B})$
Logistic	$\mathsf{Lo}(\mu,\beta)$	$f(x)=\frac{e^{-(x-\mu)/\beta}}{\beta[1+e^{-(x-\mu)/\beta}]^2}$	$x\in\mathbb{R}$	μ	$\pi^2\beta^2/3$	
Log Normal	$\mathsf{LN}(\mu,\sigma^2)$	$f(x)=\frac{1}{x\sqrt{2\pi\sigma^2}}e^{-(\log x-\mu)^2/2\sigma^2}$	$x\in\mathbb{R}_+$	$e^{\mu+\sigma^2/2}$	$e^{2\mu+\sigma^2}\left(e^{\sigma^2}-1\right)$	
Neg. Binom.	$\mathsf{NB}(\alpha,p)$	$f(x)=\binom{x+\alpha-1}{x}p^\alpha\,q^x$	$x\in\mathbb{Z}_+$	$\alpha q/p$	$\alpha q/p^2$	$(q=1-p)$
		$f(y)=\binom{y-1}{y-\alpha}p^\alpha\,q^{y-\alpha}$	$y\in\{\alpha,...\}$	α/p	$\alpha q/p^2$	$(y=x+\alpha)$
Normal	$\mathsf{No}(\mu,\sigma^2)$	$f(x)=\frac{1}{\sqrt{2\pi\sigma^2}}e^{-(x-\mu)^2/2\sigma^2}$	$x\in\mathbb{R}$	μ	σ^2	
Pareto	$\mathsf{Pa}(\alpha,\epsilon)$	$f(x)=(\alpha/\epsilon)(1+x/\epsilon)^{-\alpha-1}$	$x\in\mathbb{R}_+$	$\frac{\epsilon}{\alpha-1}$	$\frac{\epsilon^2\alpha}{(\alpha-1)^2(\alpha-2)}$	
		$f(y)=\alpha\,\epsilon^\alpha/y^{\alpha+1}$	$y\in(\epsilon,\infty)$	$\frac{\epsilon\,\alpha}{\alpha-1}$	$\frac{\epsilon^2\alpha}{(\alpha-1)^2(\alpha-2)}$	$(y=x+\epsilon)$
Poisson	$\mathsf{Po}(\lambda)$	$f(x)=\frac{\lambda^x}{x!}e^{-\lambda}$	$x\in\mathbb{Z}_+$	λ	λ	
Snedecor F	$F(\nu_1,\nu_2)$	$f(x)=\frac{\Gamma(\frac{\nu_1+\nu_2}{2})(\nu_1/\nu_2)^{\nu_1/2}}{\Gamma(\frac{\nu_1}{2})\Gamma(\frac{\nu_2}{2})}\times$ $x^{\frac{\nu_1-2}{2}}\left[1+\frac{\nu_1}{\nu_2}\,x\right]^{-\frac{\nu_1+\nu_2}{2}}$	$x\in\mathbb{R}_+$	$\frac{\nu_2}{\nu_2-2}$	$\left(\frac{\nu_2}{\nu_2-2}\right)^2\frac{2(\nu_1+\nu_2-2)}{\nu_1(\nu_2-4)}$	
Student t	$t(\nu)$	$f(x)=\frac{\Gamma(\frac{\nu+1}{2})}{\Gamma(\frac{\nu}{2})\sqrt{\pi\nu}}[1+x^2/\nu]^{-(\nu+1)/2}$	$x\in\mathbb{R}$	0	$\nu/(\nu-2)$	
Uniform	$\mathsf{Un}(a,b)$	$f(x)=\frac{1}{b-a}$	$x\in(a,b)$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$	
Weibull	$\mathsf{We}(\alpha,\beta)$	$f(x)=\alpha\beta\,x^{\alpha-1}\,e^{-\beta\,x^\alpha}$	$x\in\mathbb{R}_+$	$\frac{\Gamma(1+\alpha^{-1})}{\beta^{1/\alpha}}$	$\frac{\Gamma(1+2/\alpha)-\Gamma^2(1+1/\alpha)}{\beta^{2/\alpha}}$	