Sta 532: Homework #8

1. For specified $p_0 \in (0,1)$, $n \in \mathbb{N}$, and $\alpha \in (0,1)$, describe in detail the generalized likelihood ratio test (GLRT) of $H_0 : p = p_0$ against the two-sided alternative $H_1 : p \neq p_0$ for a single observation of $X \overset{\text{iid}}{\sim} \mathsf{Bi}(n,p)$, of size α. Precisely how would you find the rejection region $\mathcal{R}$ (perhaps using `R`)?

2. For specified $p_0 \in (0,1)$, $n \in \mathbb{N}$, and $\alpha \in (0,1)$, describe in detail the GLRT of the one-sided hypothesis $H_0 : p \geq p_0$ against $H_1 : p < p_0$ for a single observation of $X \overset{\text{iid}}{\sim} \mathsf{Bi}(n,p)$, of size α. Precisely what is the rejection region $\mathcal{R}$?

3. For $\sigma_0^2 > 0$ and $n \in \mathbb{N}$, describe in detail the GLRT of $H_0 : \sigma^2 = \sigma_0^2$ against $H_1 : \sigma^2 \neq \sigma_0^2$ for a sample of size n of $\{X_i\} \overset{\text{iid}}{\sim} \mathsf{No}(\mu, \sigma^2)$, with unknown μ, of size α. Precisely what is the rejection region $\mathcal{R}$?

4. It is generally believed that long-life light bulbs last no more than twice as long as standard light bulbs. In an experiment, the lifetimes X of a standard bulb and Y of a long-life bulb were recorded as $X = 1$, $Y = 5$. Modeling the lifetimes as exponential random variables of rates λ and θ (hence means $1/\lambda$ and $1/\theta$), respectively, construct (i) a 90% confidence interval for the ratio θ/λ, and (ii) a test of size $\alpha = 0.05$ of the hypothesis $\lambda < 2\theta$. Suggestion: Find a pivotal quantity related to θ/λ.

5. A local councilor suspects that traffic conditions have become more hazardous in Ambridge than in Borchester, so she records the numbers A and B of accidents in each place in the course of a month. Assuming that A and B are independent Poisson random variables with rates λ and θ, it is desired to construct a test of size $\alpha \approx 1/16$ of $H_0 :\ \lambda \geq \theta$ against $H_1 :\ \lambda < \theta$. Show that:

   (a) $A + B$ is Poisson with rate $(\lambda + \theta)$;

   (b) Conditional on the value n of $A + B$, that $A \sim \mathsf{Bi}(n,p)$ where $p = \frac{\lambda}{\lambda+\theta}$.

   One way to test H_0 is to *condition* on the total number $n = (A+B)$ of accidents (which itself offers no evidence about H_0) and find a rejection region $\mathcal{R}_n$ with the property that $\mathsf{P}[A \in \mathcal{R}_n :\ A + B = n] \leq \alpha$; the unconditional bound $\mathsf{P}[A \in \mathcal{R}_{A+B}] \leq \alpha$ will then follow. Carry out the test for $(A = 5,\ B = 2)$ and for $(A = 3,\ B = 0)$. You may find your solution to Problem 2 useful.

6. Find the posterior probability of H_0 in Problem 5 if λ and θ have independent exponential prior distributions with mean one, for the same two observations as before.

Last edited: March 30, 2016