If more than one choice is correct, any correct choice is correct.

1. On questions 1-6, choose for the correct choice: [1 pt] for no choice. For each wrong choice:

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   Name:
2. Businesses commonly project revenues under alternative economic scenarios. For a stylized example, inflation could be high or low and unemployment could be high or low. There are four possible scenarios, with the assumed probabilities:

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Inflation</th>
<th>Unemployment</th>
<th>Probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>high</td>
<td>high</td>
<td>0.20</td>
</tr>
<tr>
<td>2</td>
<td>high</td>
<td>low</td>
<td>0.20</td>
</tr>
<tr>
<td>3</td>
<td>low</td>
<td>high</td>
<td>0.36</td>
</tr>
<tr>
<td>4</td>
<td>low</td>
<td>low</td>
<td>0.24</td>
</tr>
</tbody>
</table>

*Hint:* Denote with $A$ the event “high inflation”, and with $B$ the event “high unemployment”.

2a [3pts] What is the probability of high inflation?

2b [4pts] What is the probability of high inflation if unemployment is high?

2c [3pts] Are inflation and unemployment independent (justify your answer)?

3. A family has two dogs (Rex and Rover) and a little boy (Russ). None of them is fond of the mailman. Given that they are outside, Rex and Rover have a 30% and a 40% chance, respectively, of biting the mailman. Russ, if he is outside, has a 15% chance of doing the same thing. Suppose only one of the three is outside when the mailman comes. Rex is outside 30% of the time, Rover 20% of the time and Russ 30% of the time.

3a [5pts] What is the probability the mailman will be bitten?

3b [5pts] If the mailman is bitten, what are the chances that Russ did it?
4. James Bond insists that his martinis be shaken, not stirred. A skeptical bartender tests Bond with 6 martinis (using six coin flips to determine which drinks to shake and which to stir). Bond errs on one and correctly identifies the other 5 before passing out. Denote with $p$ the probability that Bond can tell the difference between shaken and stirred Martinis.

4a [3pts] If $p = 0.5$, what is the probability of guessing 5 or more Martinis correctly?

4b [3pts] Find the value of $p$ such that guessing 5 out of 6 Martinis correctly is highest (justify your answer; just stating the answer is not sufficient).

   *Hint: Instead of maximizing $p(5)$ it is easier to consider $\log p(5)$.*

4c [2pts] Assume that instead of deciding initially to serve 6 Martinis, the bartender was serving Martinis until Bond guessed 5 correctly. Assuming $p = 0.5$, find the probability of erring once.

4d [2pts] Under the assumptions of 4c, find the value of $p$ such that guessing 5 out of 6 Martinis correctly is highest (justify your answer; stating the answer only is not sufficient).