1. Consider a standard normal random variable \( Z \sim N(0,1) \).

Find the following probabilities (Note: This is like homework problem 5.24)

1a. [1pt] \( P(0.7 < Z < 1.3) \)

\[
P(0.7 < Z < 1.3) = P(0 < Z < 1.3) - P(0 < Z < 0.7) = 0.4032 - 0.2580 = 0.15
\]

1b. [1pt] \( P(Z < -1.0) \)

\[
P(Z < -1.0) = 0.5 - P(0 < Z < 1.0) = 0.16
\]

Find the value \( c \) such that (Note: This is like homework problem 5.25)

1c. [1pt] \( P(Z > c) = 0.3 \)

For \( c = 0.52 \) we find \( P(0 < Z < c) = 0.2 \), i.e. \( P(Z > c) = 0.3 \)

1d. [1pt] \( P(Z > c) = 0.9 \)

For \( c = 1.28 \) we find \( P(0 < Z < c) = 0.4 \), i.e. \( P(Z > -1.28) = P(-1.28 < Z < 0) + P(0 < Z) = 0.4 + 0.5 = 0.9 \)

2. Let \( c \) be a constant and consider the density function

\[
f(y) = \begin{cases} 
  cy^2 & \text{if } 0 \leq y \leq 3 \\
  0 & \text{elsewhere}
\end{cases}
\]

Note: This problem is (almost) identical to the homework problem 5.1.

2a. [1pt] Find the value of \( c \).

\[
1 = c \int_0^3 y^2 \, dy = c \left[ \frac{y^3}{3} \right]_0^3 = c \frac{9}{3} \Rightarrow c = 1/9.
\]

Partial Credit: [1/2pt] for correct integral expression

2b. [1pt] Find the cumulative distribution function \( F(y) \).

\[
F(y) = P(Y \leq y) = c \int_0^y t^2 \, dt = c \left[ \frac{y^3}{3} \right] = y^3/27.
\]

Partial Credit: [1/2pt] for \( F(y) = P(Y \leq y) \)

2c. [1pt] Compute \( P(1 \leq y \leq 1.5) \).

\[
= F(1.5) - F(1) = (1.5^3 - 1)/27 = 0.09
\]

2d. [1pt] Find \( E(Y) \).

\[
E(y) = c \int_0^3 y \, y^2 \, dy = c \left[ \frac{y^4}{4} \right]_0^3 = 3^4/36 = 2.25
\]

Partial Credit: [1/2pt] for \( E(y) = c \int_0^3 y \, y^2 \, dy \).
3. [4pts] A random sample of \( n = 747 \) obituaries published in Salt Lake City newspapers revealed that 46\% (i.e., \( X = 344 \)) of the decedents died within the three-month period following their birthdays.

Find the probability that 46\% or more would die in that interval if deaths occurred randomly throughout the year.

What would you conclude on the basis of your answer?

Note: This is like homework problems 7.29 and 7.32 – just with different numbers.

Assuming that deaths occur randomly throughout the year \( X \) is a binomial r.v. with \( n = 747 \) and \( p = 3/12 = 0.25 \), i.e., \( X \sim Bin(n, p) \).

Partial Credit: [1pt] for stating Binomial, [1pt] for correct \( p \) and \( n \).

\[
P(X \geq 344) = P \left( \frac{X - np}{\sqrt{np(1-p)}} \geq \frac{344 - np}{\sqrt{np(1-p)}} \right) \approx P(Z \geq 13.28) = 0.0000.
\]


4. Assume \( X \) is a normal random variable with unknown mean \( \mu \) and standard deviation \( \sigma = 2.0 \) (i.e., variance \( \sigma^2 = 4.0 \)).

4a. [2pt] Find a cutoff \( c \) such that \( P(-c \leq \frac{X-\mu}{\sigma} \leq +c) = 0.95 \)

Note that \( Z = (X - \mu)/\sigma \) is standard normal \( N(0, 1) \). Find from the normal table \( P(0 < Z < 1.96) = 0.4750 \), i.e. \( P(-1.96 < Z < 1.96) = 0.95 \).

[1pt] for stating that \( Z = (X - \mu)/\sigma \) is \( N(0, 1) \).

4b*. [2pt] For appropriate coefficients \( a_1, a_2 \) and \( b_1, b_2 \), the following statement is correct:

\[
P(a_1 + a_2 X \leq \mu \leq b_1 + b_2 X) = 0.95
\]

Find \( a_1, a_2, b_1, \) and \( b_2 \).

\[
P(-c \leq \frac{X-\mu}{\sigma} \leq +c) = 0.95 \quad \Rightarrow P(X - c\sigma < \mu < X + c\sigma) = 0.95
\]

i.e. \( a_1 = -c\sigma, a_2 = 1.0, b_1 = c\sigma, \) and \( b_2 = 1.0 \).