Exercise (1)

(a) Quote results: $p(\theta|x) = N(\mu_1, \tau_1^2)$ where $\tau_1^2 = \alpha \tau_0^2$ and $\mu_1 = \alpha \mu_0 + (1 - \alpha)x$ where $\alpha = \sigma^2/(\sigma^2 + \tau_0^2)$. At $\mu_0 = 115, \tau_0 = 10, \sigma^2 = 25$ and $x = 129$ we get $\alpha = 0.2$ so that $\mu_1 = 126.2$ and $\tau_1^2 = 20$.

(b) Hence the 99% interval is $126.2 \pm 2.58 \times 20 = (114.7, 137.7)$.

(c) Under this posterior $Pr(\theta > 135|x) = 1 - \Phi((135 - \mu_1)/\tau_1) = 1 - \Phi(1.97)$ which is just less than 2.5%, so the manager’s claim appears unreasonable.

Exercise (2)

(a) $(\beta_0|Y) \sim T_{n-1}(\bar{y}, s^2/n) = T_9(11.65, 0.063^2)$. The 95% interval is $11.65 \pm 2.26 \times 0.063 = (11.51, 11.79)$.

(b) $(y_{n+1}|Y) \sim T_{n-1}(\bar{y}, s^2(1 + 1/n)) = T_9(11.65, 0.21^2)$. The 95% interval is $11.65 \pm 2.26 \times 0.21 = (11.18, 12.13)$.

(c) (A): $Pr(y_{n+1} > 12.25|Y) = 1 - F(t)$ where $t = (12.25 - 11.65)/0.21 = 2.85$, and $F$ is the cdf of the $T_9$ distribution. This $t$ value exceeds 2.82, the upper 1% point of this $T$ distribution, so the chance is indeed less than 1%.

(B): $Pr(y_{n+1} < 11.3|Y) = F(t)$ where $t = (11.3 - 11.65)/0.21 = -1.67$ which significantly exceeds the lower 1% point of $-2.82$, hence there is a really larger predictive probability of voltages lower than 11.3, and this claim is invalid.

(d) This underestimates uncertainty in prediction in two ways: first, it uses a scale factor $s^2/n$ instead of $s^2(1 + 1/n)$, and second, it assumes a normal instead of the $T_9$. 


Exercise (3)

(a) From the formula for posterior odds $(1 - \pi_0^*)/\pi_0^*$ we deduce $\pi_0^* = 1/(1 + (1 - \pi_0)B_{1,0}/\pi_0)$. The values here imply $\pi_0^* = 1/(1 + 3.53) = 0.22$.

(b) We can rearrange the above equation to give $\pi_0 = 1/(1 + (1 - \pi_0^*)/(B_{1,0}\pi_0^*))$. Hence in this case we get $\pi_0 = 1/(1 + 0.97/(0.03 \times 5.8)) = 0.15$.

(c) We have $\hat{\mu} = \max\{0, y/n - b\} = 0.0269$ in this case of $y = 2725, n = 3600$ and $b = 0.73$.

Then
\[
B = (n(b + \hat{\mu}))^b \exp(-n(b + \hat{\mu}))/\{(nb)^b \exp(-nb)\} = (1 + \hat{\mu}/b)^b \exp(-\eta\hat{\mu})
\]
which gives 5.86.

(d) Since $\pi_0^* = 1/(1 + (1 - \pi_0)B_{1,0}/\pi_0)$ then $B_{1,0} \leq B$ implies $\pi_0^* \geq 1/(1 + (1 - \pi_0)B/\pi_0)$. Here therefore we have $\pi_0^* \geq 1/(1 + 5.86) = 0.146$.

(e) Whatever prior you might assume, an initial 50:50 position on $H_0$ can never reduce past a posterior probability of about 15%, or odds of about 6:1 against. This is a fairly high lower limit and suggests that this data is simply not persuasive about the existence of a supernova.

(f) A $p$-value is NOT a probability of the null hypothesis, so the astronomer friend is simply wrong. And if it is incorrectly interpreted as such, it is misleading because the 3% level is a factor of 5 lower than the lower bound on the posterior probability as computed above. The $p$-value appears to overstate evidence against the null, as usual.
Exercise (4)

(a) Unclear: Small numbers of women and some men seem to have lower salaries that women, some higher. The graph can mislead because it hides the effects of other factors: for example, maybe all the more highly paid men are also all highly educated and in higher management levels.

(b) These two models have the same number of predictors (just one in each model) so fit.2 is better as it explains a far higher proportion of variation $R^2$.

(c) fit.3 has one more parameter so “loses” one more degree of freedom.

(d) Yes, it explains about 95% of total variation in the salary data, and most, if not all, of the estimated parameters are highly significant – the only marginal one is the Sex parameter.

(e) That log salaries are, on average, 0.0228 higher for women than for men, when adjusted for years in service, education and management level as implied by the model. There is a great deal of uncertainty however – a 95% interval for the mean difference on the log scale (about 2 s.e.s either side of 0.0228) is about $-0.0034$ to $0.0490$, which includes negative values, and so it looks as though there may be no difference detectable on this data alone. This essentially agrees with the graph which is not clear either way.

(f) The significant estimate of 0.092 for Ed1 indicates that, relative to a high-school education (level 0 of Ed) the Bac. Degree (level 1) is associated with higher average salaries: the average difference of 0.092 on the log scale implies $\exp(0.092) = 1.1$, so about a 10% higher levels on average on the actual salary scale.

(g) $\exp(0.0295) = 1.03$ suggesting salaries are generally only 3% higher, on average, for employees with a graduate degree relative to those with only high school educations. This implies that, with all the other factors taken into account, individuals with graduate degrees earn significantly lower salaries on average than those with only Bac. degrees. The graphs help interpret this as being due to multicollinearity – by linking across frames, we infer that employees with higher degrees tend to be in higher management, and having fitted the management factor we find that there is downward correction now needed for graduate degree holders.