1. The first problem concerned the Bayes Ball algorithm.
   a. \( P(E) \)  
      Bold circles are the query nodes. Shaded circles are the evidence.

   
   
   N_p = \{E\}
   N_e = {} 

   b. \( P(E|H) \) 

   
   
   N_p = \{A,B,D,E,H\}
   N_e = \{H\} 

   c. \( P(F) \) 

   
   
   N_p = \{F,C,A\}
   N_e = {}
d. $P(F|A)$

$N_p = \{F, C\}$

$N_e = \{A\}$

e. $P(B|D, G)$ Almost everyone missed a part of this problem (the bounce back up to C from G.).

$N_p = \{B, D, A, C, G\}$

$N_e = \{D, G\}$
2. Expert System Problem:

2.a construct junction tree.
(i) moralize

(ii) triangulate (done)
(iii) junction graph:

(iv) junction tree (Kruskal's algorithm)
(v) Assign potentials
P(C) to CGU
P(G) to CGU
P(W|CG) to CGW
P(U|CG) to CGU
P(T|U) to CT

The separators are all 1.0
We don't actually have to propagate prior to using the junction tree.

2.b

Separator between CGU and CGW: \( \Phi_*(CG) \)
Separator between CGW and CT: \( \Phi_*(C) \)

Propagation:
Absorption from CGU to CGW
\( \Phi_*(CG) = \)
\[
\begin{array}{cccc}
0.002975 & 0.049875 & 0.0273 & 0.006175 \\
0.01435 & 0.266 & 0.003575 & 0.006175 \\
\end{array}
\]

Rows that have a value of zero are suppressed
Absorption from CGW to CT
\begin{align*}
\Phi_{S}^{*}(C) = & \begin{array}{ll}
C=p & 0.024094 \\
C=a & 0.0224218
\end{array} \\
\Phi_{C}^{*}(CT) = & \begin{array}{ll}
CT & \begin{array}{ll}
C=p & 0.0069873 \\
C=a & 0.0219733
\end{array} \\
G=p & 0.0171067 \\
G=a & 0.0004484
\end{array}
\end{align*}

Absorption from CT to CGW
\begin{align*}
\Phi_{S}^{*}(C) = & \begin{array}{ll}
C=p & 0.024094 \\
C=a & 0.0224218
\end{array} \\
\Phi_{C}^{*}(CGW) = & \begin{array}{llll}
C=p & G=p & G=a & 0.0026478 \\
C=a & G=p & G=a & 0.0214463 \\
& & & 0.022113 \\
& & & 0.00030875
\end{array}
\end{align*}

Absorption from CGW to CGU
\begin{align*}
\Phi_{S}^{*}(C) = & \begin{array}{ll}
C=p & 0.0026478 \\
C=a & 0.0214463 \\
G=p & 0.022113 \\
G=a & 0.00030875
\end{array} \\
\Phi_{C}^{*}(CGU) = & \begin{array}{llll}
C=p & G=p & G=a & 0.0569216 \\
C=a & G=p & G=a & 0.4610535 \\
& & & 0.4753874 \\
& & & 0.006637537
\end{array}
\end{align*}

Note that the W=p row of CGU and the U=v row of CGU now have the same joint distribution over C and G.

Calculate \( P(C|W=p, U=v) \) using the potential for CT.
\[
0.5179751 = \frac{(D157+D158)}{SUM(D157:E158)}
\]
Calculate \( P(G|W=p, U=v) \) using the potential for CGU.
\[
0.5323089 = \frac{(D150+F150)}{SUM(D150:G150)}
\]

Calculate the joint distribution \( P(CG|W=p, U=v) \) by normalizing \( \Phi_{C}(CGU) \)
\begin{align*}
\Phi_{C}(CGU) = & \begin{array}{llll}
C=p & G=p & G=a & 0.0569216 \\
C=a & G=p & G=a & 0.4610535 \\
& & & 0.4753874 \\
& & & 0.006637537
\end{array}
\end{align*}

Note that C and G are anti-correlated.

2.c Start with the old junction tree (part 2.b)
Now observe that T = p
Multiply in evidence into \( \Phi_{C}(CT) \)
\begin{align*}
\Phi_{C}(CT) = & \begin{array}{ll}
CT & \begin{array}{ll}
C=p & 0 \\
C=a & 0
\end{array} \\
G=p & 0.0171067 \\
G=a & 0.0004484
\end{array}
\end{align*}
Absorb evidence from CT to CGW and stop.

\[ \Phi_S^*(C) = \begin{pmatrix} 0.0171067 \\ 0.0004484 \end{pmatrix} \]

\[ \Phi_S^*(C)/\Phi_S(C) = \begin{pmatrix} 0.71 \\ 0.02 \end{pmatrix} \]

<table>
<thead>
<tr>
<th>( \Phi_C(CGW) )</th>
<th>( C=p )</th>
<th>( C=a )</th>
<th>( G=p )</th>
<th>( G=a )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Old value</td>
<td>0.0026478</td>
<td>0.0214463</td>
<td>0.022113</td>
<td>0.00030875</td>
</tr>
</tbody>
</table>

\[ \Phi_C*(CGW) = \begin{pmatrix} 0.0018799 \\ 0.0152268 \\ 0.0004423 \\ 0.000006175 \end{pmatrix} \]

Calculate \( P(C|W=p, U=v) \) using the potential for CGW.

\[ 0.9744557 \]

Calculate \( P(G|W=p, U=v) \) using the potential for CGW.

\[ 0.1322779 \]

2.d Start with the old junction tree (part 2.b)

Absorb evidence from CT to CGW and stop.

\[ \Phi_S^*(C) = \begin{pmatrix} 0.0069873 \\ 0.0219733 \end{pmatrix} \]

\[ \Phi_S^*(C)/\Phi_S(C) = \begin{pmatrix} 0.29 \\ 0.98 \end{pmatrix} \]

\[ \Phi_C(CGW) = \begin{pmatrix} 0.007678 \\ 0.0062194 \\ 0.0216707 \end{pmatrix} \]

Calculate \( P(C|W=p, U=v) \) using the potential for CGW.

\[ 0.241268 \]

Calculate \( P(G|W=p, U=v) \) using the potential for CGW.

\[ 0.7747977 \]