The Correct Proof for Intersection

Intersection: If a JPD is positive, for all disjoint $W, X, Y, Z$,
$I(X, W|Z \cup Y) \land I(X, Y|Z \cup W) \Rightarrow I(X, Y \cup W|Z)$.

Proof:

$$I(X, W|Z \cup Y) \Rightarrow \quad (\text{EQ 1})$$

$$P(XYZ)P(WYZ) = P(WXYZ)P(YZ)$$

$$I(X, Y|Z \cup W) \Rightarrow \quad (\text{EQ 2})$$

$$P(WXZ)P(WYZ) = P(WXYZ)P(WZ)$$

Divide equation (2) by equation (1)

$$\frac{P(WXZ)}{P(XYZ)} = \frac{P(WZ)}{P(YZ)} \quad (\text{EQ 3})$$

Sum $W$ out of both sides of (3).

$$\sum_w \frac{P(WXZ)}{P(XYZ)} = \sum_w \frac{P(WZ)}{P(YZ)} \quad (\text{EQ 4})$$

$$\frac{P(XZ)}{P(YZ)} = \frac{P(YZ)}{P(YZ)} \quad (\text{EQ 5})$$

Rearrange the factors of (5):

$$\frac{P(XZ)}{P(Z)} = \frac{P(XYZ)}{P(YZ)} \quad (\text{EQ 6})$$

Definition of conditional probability:

$$P(X|Z) = P(X|YZ) \quad (\text{EQ 7})$$

Divide both sides of (1) by $P(YZ)P(WYZ)$

$$\frac{P(XYZ)}{P(YZ)} = \frac{P(WXYZ)}{P(WYZ)} \quad (\text{EQ 8})$$

Definition of conditional probability:

$$P(X|YZ) = P(X|WYZ) \quad (\text{EQ 9})$$

Substitute (7) into (9):

$$P(X|Z) = P(X|WYZ) \quad (\text{EQ 10})$$

(10) is the desired consequent:

$$P(X|Z) = P(X|WYZ) \Rightarrow I(X, W \cup Y|Z)$$