The BEST Award for Student Research

2011-12 BEST Award Honorable Mention

KATE YUAN
Persistence, Leverage Effects, Jumps and Heavy-Tails in International Equity Markets
Persistence, Leverage Effects, Jumps and Heavy-tails in International Equity Markets

Kate Yuan
Trinity College, Duke University ’12

March 31, 2012
Acknowledgements

I would first and foremost like to thank Professor Ma for his guidance, patience and encouragement throughout my senior year. I could not have accomplished this project without his help. I would also like to extend my gratitude to Professor Stangl, Professor Reiter, and my STA190 classmates for valuable advice and feedback. In addition, I would especially like to thank Jouchi Nakajima, whose work I based my project on, for his insights and constant support. Finally, I would like to thank Professor Rasiel and Professor Levonmaa in the Economic Department for helping me collect the data used in the project, and Professor Tauchen for sharing his knowledge of the financial markets.
Abstract
Understanding how equity returns change over time is of great use in understanding macroeconomy as well as the dynamics in the global equity markets. Time-varying volatility modeled through the process of conditional variance proves to be an effective measure of the changes in financial returns. In particular, the ARCH (Autoregressive Conditional Heteroskasticity) class volatility models are shown to adequately capture the persistence in volatility. Among the various extensions of the ARCH/GARCH family, EGARCHJt (Exponential Generalized ARCH with jumps and heavy-tails) is a model that incorporates heavy-tailness and asymmetric impact from returns in the estimation process.

In this paper, we explore the volatility behavior in Emerging Markets (EM) and how it is different from that in the U.S., as EM gain increasing attention from global investors. We apply the EGARCHJt model to a broad set of data that comprises both U.S. and EM equities and compare volatility behaviors across countries in these markets. The results suggest that EM tends to have lower level of persistence, less degree of leverage effects, more jumps and heavier tails than the U.S. market.

Keywords: GARCH; Bayesian Analysis; Heavy-tails; Jumps; Emerging Markets; Equities; Markov Chain Monte Carlo
1 Introduction

Engle (1982), in his ARCH (Autoregressive Conditional Heteroskedasticity) model, distinguishes conditional variance from unconditional variance by allowing the former to be a function of past residuals. Bollerslev (1986) generalizes ARCH (GARCH) so that the conditional variance is a constant plus a weighted average (with positive weights) of past residuals as well as lagged conditional variance. Since then, there have been numerous extensions on the family of ARCH/GARCH (see Bollerslev (2007) for a glossary to ARCH).

A simple GARCH(1, 1) with normal errors is formulated as:

\[
y_t = \epsilon_t \sigma_t, \quad \epsilon_t \sim N(0, 1)
\]

\[
\sigma_t^2 = \omega + \beta \sigma_{t-1}^2 + \alpha y_{t-1}^2
\]

where \( y_t \) is the random process (or equity returns in the context of this paper) and \( \sigma_t^2 \) is the conditional variance. The model specifies that \( \omega > 0, 1 > \beta > 0 \) and \( \alpha > 0 \) to ensure non-negativity of the conditional variance and stationarity of the process.

While GARCH is shown to successfully capture the volatility clustering in equity returns, i.e., the fact that large changes tend to be followed by large changes and small changes followed by small changes (Mandelbrot (1963)), there are certain limitations to the model. We list three that are relevant in our analysis below. First and foremost, GARCH is formed in a way that only the magnitude but not the sign of the lagged residuals \( y_{t-1} \) enters the conditional variance. In other words, it assumes that negative past returns effect current period’s volatility the same amount as do positive past returns. However, as observed by practitioners and researchers, a negative shock to financial time-series is likely to cause volatility to rise more than a positive shock of the same magnitude; such asymmetry is called “leverage effects” in equity markets (Brooks (2008)). Secondly, GARCH is estimated under the assumption that errors are conditionally normal. Yet the standardized residuals appear to have fatter tails than a Normal distribution in many empirical applications (Bollerslev (2007)). Finally, the non-negativity constraints imposed on the coefficients make it difficult to estimate the model (Nelson (1991)).

In modern financial econometrics, returns are often modeled in a way that implies the price series is from a continuous-time diffusive process (Huang and Tauchen (2005)). However, a number of empirical and theoretical studies suggest that financial variables sometimes exhibit significant discontinuities, so-called “jumps”, which have substantial impact on portfolio and risk management, and provide better explanation for the excess skewness and kurtosis of return distributions (Lee and Mykland (2008)). Therefore, in addition to compensating for the GARCH drawbacks discussed in the previous paragraph, it is also useful to include a jump component when modeling time-series volatility.

In this paper, we employ the EGARCHJt model proposed by Nakajima (2008), one particular extension of the ARCH/GARCH family, to account for the leverage
effects, jumps and heavy-tails of equity returns described above. (See the work of Nelson (1991) for more details on Exponential Generalized ARCH.) The model is estimated through Bayesian inference with MCMC techniques developed by Nakajima (2008). His application to daily returns of S&P500 and NASDAQ successfully reflects the volatility characteristics in the U.S. In this paper, we apply the model not only to U.S. equities but also to equity returns in the Emerging Markets. Our goal is to distinguish volatility behaviors in developed and developing markets, as well as compare volatility features across individual countries/clusters in the EM.

The rest of the paper is organized as follows. In Section 2, we provide an overview of the EGARCHJt model. In Section 3, we give the detailed derivation and MCMC algorithm. In Section 4, we use simulations to illustrate the Bayesian estimation process and investigate the general properties of the model. In Section 5, we discuss some basic features of the data. In Section 6, we apply the EGARCHJt model to fifteen series of weekly equity data, including the Russell 3000 Index, the overall EM Growth Index and thirteen countries/clusters in the EM. Finally, Section 7 concludes the paper.

2 The EGARCHJt Model

Following the derivation and notations of Nakajima (2008), the EGARCH(1,1) model with jumps and heavy-tails is formulated as:

\[
y_t = k_t \gamma_t + \sqrt{\lambda_t} \epsilon_t, \quad \epsilon_t \sim N(0, 1) \quad (3)
\]

\[
\log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \theta \left( \frac{y_{t-1}}{\sigma_{t-1}} \right) + \alpha \left( \frac{|y_{t-1}|}{\sigma_{t-1}} - \zeta \right) \quad (4)
\]

where \(y_t\) is the equity return for period \(t\) and \(\{\sigma_t^2\}_{t=1}^n\) is the conditional variance of the process. Below is the model specification:

In the first part of equation (3), \(k_t \gamma_t\) represents the jump component in the model. \(k_t\) is the jump size that measures the magnitude of sudden excess returns (of either sign), and is normally distributed with standard deviation \(\delta\). The presence of a jump for the period \(t\) is indicated by the jump flag, \(\gamma_t\), defined as a Bernoulli random variable. \(\delta\) and \(\kappa\) are unknown hyper-parameters to be estimated.

\[
k_t \sim N(0, \delta^2)
\]

\[
\pi(\gamma_t = 1) = \kappa, \quad \pi(\gamma_t = 0) = 1 - \kappa, \quad 0 < \kappa < 1
\]

In the second part of equation (3), \(\sqrt{\lambda_t} \epsilon_t\) is the measurement error and is constructed to follow a heavy-tailed Student-t distribution with unknown degrees of freedom \(\nu\) by letting

\[
\lambda_t^{-1} \sim \text{Gamma}(\nu/2, \nu/2)
\]

See the derivation for the Student-t distribution in Appendix A. Adding the \(\sqrt{\lambda_t}\) term makes the marginal distribution of \(y_t\) more spread out towards the tails than
the GARCH(1,1) with a normal error $\epsilon_t$. When $\lambda_t \equiv 1$ for all $t$, equation (3) becomes $y_t = k_t \gamma_t + \epsilon_t \sigma_t$ and the model reduces to EGARCH with jumps and normal errors.

In equation (4), the coefficient $\beta$ is constrained to be $0 < \beta < 1$ to ensure stationarity. Intuitively, $\beta$ measures how dependent the log conditional variance of this period is on the lagged log conditional variance. $\beta$ close to 1 is an evidence for the high persistence in financial volatility.

The coefficient $\theta$ allows the sign of $y_{t-1}$ to impact log $\sigma_t$, and measures the “leverage effects” described in the Introduction. The second half of equation (4) can be written as

\[
\theta \left( \frac{y_{t-1}}{\sigma_{t-1}} \right) + \alpha \left( \frac{|y_{t-1}|}{\sigma_{t-1}} - \zeta \right) = (\theta + \alpha) \left( \frac{y_{t-1}}{\sigma_{t-1}} \right) - \alpha \zeta \quad \text{if } y_{t-1} > 0
\]

\[
\theta \left( \frac{y_{t-1}}{\sigma_{t-1}} \right) + \alpha \left( \frac{y_{t-1}}{\sigma_{t-1}} - \zeta \right) = (-\theta + \alpha) \left( \frac{-y_{t-1}}{\sigma_{t-1}} \right) - \alpha \zeta \quad \text{if } y_{t-1} < 0
\]

When $\theta < 0$, $(-\theta + \alpha) > (\theta + \alpha)$ and therefore a negative $\theta$ would imply an increase in conditional volatility following a previous drop in stock return.

Finally, $\zeta = E(|\epsilon_1|)$ where $\epsilon_t$ following a Student-t distribution with degrees of freedom $\nu$. Let $z_t = y_t/\sigma_t$ for all $t$ to represent the scaled returns. We can re-write Equation (4) as

\[
\log \sigma_t^2 = \omega + \beta \log \sigma_{t-1}^2 + \theta z_{t-1} + \alpha (|z_{t-1}| - \zeta)
\]

To see that the term $\alpha (|z_{t-1}| - \zeta)$ represents a magnitude effect of the lagged residuals discussed in Section 1, assume for the moment that $\theta = 0$ and $\alpha > 0$. From the specification of $\sqrt{\lambda_t} \epsilon_t$ above, we know that $z_t$ also follows a Student-t distribution with $\nu$ degrees of freedom conditioning on $k_t$, $\gamma_t$ and $\sigma_t^2$. Thus, the innovation in $\log \sigma_t^2$ is positive when the magnitude of $z_{t-1}$ is larger than its expected value, and negative when smaller than its expected value (Nelson (1991)).

For simplicity, the unconditional variance for the initial period is assumed to be $\log \sigma_1^2 = \omega / (1 - \beta)$.

3 Methodology

3.1 Derivation of Likelihood Function

In financial econometrics, the coefficients for GARCH-type volatility models are often estimated by using maximum likelihood. In this paper, the EGARCHJt model is estimated through Bayesian inference using the MCMC algorithm.

Let $y = \{y_t^2\}_{t=1}^n$, $\gamma = \{\gamma_t\}_{t=1}^n$, $\lambda = \{\lambda_t\}_{t=1}^n$ and $\vartheta = (\omega, \beta, \theta, \alpha)$. We first derive the log likelihood function.
In equation (3), we specify that
\[ k_t \sim N(0, \delta^2), \quad \epsilon_t \sim N(0, 1) \]

After marginalizing over \( k_t \), the conditional distribution of \( y_t \) follows the Normal distribution below
\[ y_t | \gamma_t, \lambda_t, \sigma_t^2 \sim N(0, \gamma_t^2 \delta^2 + \lambda_t \sigma_t^2) \]

Since \( \gamma_t \) is a binary variable, the variance of the Normal distribution above can be further simplified to
\[ y_t | \gamma_t, \lambda_t, \sigma_t^2 \sim N(0, \gamma_t \delta^2 + \lambda_t \sigma_t^2) \]

The likelihood function is thus
\[ L \propto \prod_{t=1}^n \frac{1}{\sqrt{\gamma_t \delta^2 + \lambda_t \sigma_t^2}} \exp \left\{ -\frac{y_t^2}{2(\gamma_t \delta^2 + \lambda_t \sigma_t^2)} \right\} \] (5)

Then we transform the likelihood function to the log scale up to a constant
\[ \log L = \sum_{t=1}^n \log \frac{1}{\sqrt{\gamma_t \delta^2 + \lambda_t \sigma_t^2}} \exp \left\{ -\frac{y_t^2}{2(\gamma_t \delta^2 + \lambda_t \sigma_t^2)} \right\} \]
\[ = \sum_{t=1}^n \log(\gamma_t \delta^2 + \lambda_t \sigma_t^2)^{-\frac{1}{2}} - \frac{y_t^2}{2(\gamma_t \delta^2 + \lambda_t \sigma_t^2)} \]
\[ = \sum_{t=1}^n -\frac{1}{2} \log(\gamma_t \delta^2 + \lambda_t \sigma_t^2) - \frac{y_t^2}{2(\gamma_t \delta^2 + \lambda_t \sigma_t^2)} \]

### 3.2 Bayesian Inference and MCMC algorithm

Our MCMC algorithm for the EGARCHJt is designed following Nakajima (2008) and is illustrated in this section.

1. Initialize \( \theta \), \( \gamma \), \( \delta \), \( \kappa \), \( \lambda \) and \( \nu \), and generate iteratively S passes for all the parameters. A single pass is decomposed as follows
   \[ \theta^{(s+1)} \sim \pi(\theta | \delta^{(s)}, \gamma^{(s)}, \lambda^{(s)}, y) \]
   \[ \delta^{(s+1)} \sim \pi(\delta | \theta^{(s+1)}, \gamma^{(s)}, \lambda^{(s)}, \kappa^{(s)}, y) \]
   \[ \gamma^{(s+1)} \sim \pi(\gamma | \theta^{(s+1)}, \delta^{(s+1)}, \lambda^{(s)}, \kappa^{(s)}, y) \]
   \[ \kappa^{(s+1)} \sim \pi(\kappa | \theta^{(s+1)}, \delta^{(s+1)}, \gamma^{(s+1)}, \lambda^{(s)}, y) \]
   \[ \lambda^{(s+1)} \sim \pi(\lambda | \theta^{(s+1)}, \delta^{(s+1)}, \gamma^{(s+1)}, \nu^{(s)}, y) \]
   \[ \nu^{(s+1)} \sim \pi(\nu | \lambda^{(s+1)}, y) \]

2. Sample \( \theta | \delta, \gamma, \lambda, y \).

Since the posterior distribution of \( \theta \) does not have a closed-form distribution, \( \theta \) is sampled by the Metropolis-Hastings algorithm from a Normal approximation of its
conditional posterior distribution $\pi(\varnothing|\delta, \gamma, \lambda, y)$.

To obtain the proposal distribution for the Metropolis-Hastings, we first find $\tilde{\varnothing}$ that maximizes the log posterior density $\pi(\varnothing|\delta, \gamma, \lambda, y)$, and approximate the proposal mean and variance from second-order Taylor expansion of $\pi(\varnothing|\delta, \gamma, \lambda, y)$ around $\tilde{\varnothing}$. Let $f(\varnothing) = \log \pi(\varnothing|\delta, \gamma, \lambda, y)$, we have

$$\pi(\varnothing|\delta, \gamma, \lambda, y) = \exp f(\varnothing) \approx \exp \left\{ f(x) + \frac{f'(x)}{2!} (\varnothing - x)^2 \right\} \bigg|_{\varnothing = \tilde{\varnothing}}$$

$$= \exp \left\{ \frac{1}{2} f''(x) + \varnothing f'(x) - f''(x) \right\} \bigg|_{\varnothing = \tilde{\varnothing}} \propto \exp \left\{ \frac{1}{2} f''(x) \right\} \bigg|_{\varnothing = \tilde{\varnothing}}$$

which is the kernel of a Normal distribution $(\mu, \Sigma)$ where

$$\mu = x - \frac{f'(x)}{f''(x)} \bigg|_{\varnothing = \tilde{\varnothing}} \quad \text{and} \quad \Sigma = -\frac{f''(x)}{f''(x)} \bigg|_{\varnothing = \tilde{\varnothing}}$$

Since $\tilde{\varnothing}$ approximately maximizes the log posterior density, $f'(x) \approx 0$. For the simplicity of calculation, we remove the second term in the expression of $\mu$ and therefore obtain the Normal proposal distribution $N(\tilde{\varnothing}, \tilde{\Sigma})$ for $\varnothing$ where

$$\tilde{\mu} = \tilde{\varnothing}, \quad \tilde{\Sigma}^{-1} = -\frac{\partial^2 \log \pi(\tilde{\varnothing}|\varnothing, \delta, \gamma, \lambda, y)}{\partial \varnothing^2} \bigg|_{\varnothing = \tilde{\varnothing}}$$

Empirically, $\tilde{\varnothing}$ is approximated through an iterative procedure where we fix all but one parameter in $\tilde{\varnothing}$ at a time, and solve for its maximum. In other words, we find $\tilde{\omega}, \tilde{\beta}, \tilde{\theta}$ and $\tilde{\alpha}$ that maximize their log posterior densities respectively. In addition, prior independence is assumed so that $\pi(\varnothing) = \pi(\omega)\pi(\beta)\pi(\theta)\pi(\alpha)$. See Appendix B for step-by-step derivation for $\tilde{\varnothing}$.

We then draw a candidate $\varnothing^*$ for Metropolis-Hastings from $N(\tilde{\mu}, \tilde{\Sigma})$. Let $\varnothing^{(s)}$ denote the current point of $\varnothing$ and $q$ denote the proposal density, we accept the candidate $\varnothing^*$ with probability

$$\alpha(\varnothing^{(s)}, \varnothing^*|\delta, \gamma, \lambda, y) = \min \left\{ \frac{\pi(\varnothing^*|\delta, \gamma, \lambda, y)q(\varnothing^{(s)}|\delta, \gamma, \lambda, y)}{\pi(\varnothing^{(s)}|\delta, \gamma, \lambda, y)q(\varnothing^*|\delta, \gamma, \lambda, y)}, 1 \right\}$$

If the candidate $\varnothing^*$ is rejected, we pass the current value $\varnothing^{(s)}$ to the next draw $\varnothing^{(s+1)}$. 8
3. Sample \((\delta, \gamma)|\vartheta, \lambda, \kappa, y\).
   a) Sample \(\delta|\vartheta, \lambda, \kappa, y\).
   
   The joint distribution of \((\delta, \gamma)\) is given by
   \[
   \pi(\delta, \gamma|\vartheta, \lambda, \kappa, y) \propto \pi(\delta) \pi(\gamma|\kappa) f(y|\vartheta, \gamma, \lambda, \kappa)
   \]
   \[
   \propto \pi(\delta) \prod_{t=1}^{n} \kappa^{\gamma_t} (1 - \kappa)^{1 - \gamma_t} \frac{1}{\sqrt{\gamma_t \delta^2 + \lambda_t \sigma_t^2}} \exp \left\{ -\frac{y_t^2}{2(\gamma_t \delta^2 + \lambda_t \sigma_t^2)} \right\}
   \]
   
   To sample \(\delta\), we marginalize this joint posterior distribution over \(\gamma\).
   \[
   \pi(\delta|\gamma = 1, \vartheta, \lambda, \kappa, y) \propto \pi(\delta) \prod_{t=1}^{n} \frac{\kappa}{\sqrt{\delta^2 + \lambda_t \sigma_t^2}} \exp \left\{ -\frac{y_t^2}{2(\delta^2 + \lambda_t \sigma_t^2)} \right\}
   \]
   \[
   \pi(\delta|\gamma = 0, \vartheta, \lambda, \kappa, y) \propto \pi(\delta) \prod_{t=1}^{n} \frac{1 - \kappa}{\sqrt{\lambda_t \sigma_t^2}} \exp \left\{ -\frac{y_t^2}{2(\lambda_t \sigma_t^2)} \right\}
   \]
   
   Hence, by combining the two equations above, the marginalized conditional posterior distribution of \(\delta\) is formed as
   \[
   \pi(\delta|\vartheta, \lambda, \kappa, y) \propto \pi(\delta) \prod_{t=1}^{n} \left[ \frac{\kappa}{\sqrt{\delta^2 + \lambda_t \sigma_t^2}} \exp \left\{ -\frac{y_t^2}{2(\delta^2 + \lambda_t \sigma_t^2)} \right\} + \frac{1 - \kappa}{\sqrt{\lambda_t \sigma_t^2}} \exp \left\{ -\frac{y_t^2}{2(\lambda_t \sigma_t^2)} \right\} \right]
   \]
   
   We specify the prior for \(\delta^2\) as Inverse-Gamma\((m_\alpha, m_\beta)\) and therefore have
   \[
   \pi(\delta) \propto (\delta^2)^{-m_\alpha - 1} \exp \left\{ -\frac{m_\beta}{\delta^2} \right\} \left| \frac{d\delta^2}{d\delta} \right| = 2(\delta^{-2m_\alpha - 1}) \exp \left\{ -\frac{m_\beta}{\delta^2} \right\}
   \]
   Since this posterior does not have a closed-form distribution, we sample \(\delta\) by the M-H algorithm with a Normal proposal distribution where the proposal mean and variance are constructed using the same technique in Step 2.

   b) Sample \(\gamma|\vartheta, \delta, \kappa, \lambda, y\).
   
   The posterior distribution of \(\gamma\) follows a Bernoulli distribution. We sample \(\gamma_t\) for \(t = 1, ..., n\) using the probability mass function of its posterior density
   \[
   \pi(\gamma = 1|\vartheta, \delta, \kappa, y) = \frac{\kappa}{\sqrt{\delta^2 + \lambda_t \sigma_t^2}} \exp \left\{ -\frac{y_t^2}{2(\delta^2 + \lambda_t \sigma_t^2)} \right\}
   \]
   \[
   \pi(\gamma = 0|\vartheta, \delta, \kappa, y) = \frac{1 - \kappa}{\sqrt{\lambda_t \sigma_t^2}} \exp \left\{ -\frac{y_t^2}{2(\lambda_t \sigma_t^2)} \right\}
   \]
4. Sample $\kappa|\vartheta, \delta, \gamma, \lambda, y$.
We specify the conjugate-prior for $\kappa$ as $Beta(n_{\alpha 0}, n_{\beta 0})$, and derive the posterior distribution below

$$
\pi(\kappa|\vartheta, \delta, \gamma, \lambda, y) \propto \pi(\kappa)\pi(\gamma|\kappa)f(y|\vartheta, \delta, \gamma, \lambda, \kappa) \\
\propto \kappa^{n_{\alpha 0}}(1 - \kappa)^{n_{\beta 0}} \prod_{t=1}^{n} \pi(\gamma_t|\kappa) \\
\propto \kappa^{n_{\alpha 0}}(1 - \kappa)^{n_{\beta 0}K\sum_{t: \gamma_t=1} - (1 - \kappa)\sum_{t: \gamma_t=0}} \\
\propto \kappa^{n_{\alpha 0} + \sum_{t: \gamma_t=1}}(1 - \kappa)^{n_{\beta 0} + \sum_{t: \gamma_t=0}}
$$

Let $n_1$ and $n_0$ denote the count of $\gamma_t = 1$ and $\gamma_t = 0$ respectively, the posterior distribution is as follows

$$\kappa \sim Beta(n_{\alpha 0} + n_1, n_{\beta 0} + n_0)$$

5. Sample $(\lambda, \nu)|\vartheta, \delta, \gamma, y$.

a) Sample $\lambda|\vartheta, \delta, \gamma, \nu, y$.
The joint posterior distribution of $(\lambda, \nu)$ is given by

$$
\pi(\lambda, \nu|\vartheta, \delta, \gamma, \nu, y) \propto \pi(\nu)\pi(\lambda|\nu)f(y|\vartheta, \delta, \gamma, \lambda, \nu) \\
\propto \pi(\nu)\prod_{t=1}^{n} \frac{(\frac{\nu}{2})^{\frac{n_2}{2}}}{\Gamma(\frac{n_2}{2})} \lambda_t^{-\frac{n_2}{2} - 1} \exp \left\{-\frac{\nu}{2\lambda_t} \right\} \prod_{t=1}^{n} \frac{1}{\sqrt{\gamma_t\delta^2 + \lambda_t\sigma^2_t}} \exp \left\{-\frac{y_t^2}{2(\gamma_t\delta^2 + \lambda_t\sigma^2_t)} \right\}
$$

We sample $\lambda_t$ from its full conditional distribution

$$
\pi(\lambda_t|\vartheta, \delta, \nu, y) \propto \lambda_t^{-\frac{n_2}{2} - 1} \exp \left\{-\frac{\nu}{2\lambda_t} \right\} \frac{1}{\sqrt{\gamma_t\delta^2 + \lambda_t\sigma^2_t}} \exp \left\{-\frac{y_t^2}{2(\gamma_t\delta^2 + \lambda_t\sigma^2_t)} \right\}
$$

by the Metropolis-Hastings algorithm where we draw the candidate $\lambda_t^*$ from proposal distribution $Inverse-Gamma(\eta, \eta)$. The tuning parameter $\eta$ is set to be proportional to the value of $\nu$ depending on the M-H acceptance rate.

b) Sample $\nu|\lambda$.
Finally, the conditional posterior distribution for $\nu$ given $\lambda$ is

$$
\pi(\nu|\lambda) \propto \pi(\nu)\prod_{t=1}^{n} \frac{(\frac{\nu}{2})^{\frac{n_2}{2}}}{\Gamma(\frac{n_2}{2})} \lambda_t^{-\frac{n_2}{2} - 1} \exp \left\{-\frac{\nu}{2\lambda_t} \right\} \\
\propto \pi(\nu)\left(\frac{\nu}{2}\right)^{\frac{n_2}{2} + \sum_{t}^{n}} \prod_{t=1}^{n} \left\{ \lambda_t^{-\frac{n_2}{2}} \exp \left\{-\frac{\nu}{2\lambda_t} \right\} \right\} \\
\propto \pi(\nu)\left(\frac{\nu}{2}\right)^{\frac{n_2}{2}} \prod_{t=1}^{n} (\lambda_t^{-\frac{n_2}{2}}) \left\{ \exp \left\{-\frac{\nu}{2} \sum_{t=1}^{n} \lambda_t^{-1} \right\} \right\}
$$

Again, we sample $\nu$ by Metropolis-Hastings with a Normal proposal distribution constructed using the technique in Step 2.
4 Simulation

4.1 Priors

This section shows the estimation procedure for the EGATHCJt model using simulated data. We generate 3,000 new observations from the model given by equation (3) and (4) with $\omega = -0.02$, $\beta = 0.95$, $\theta = -0.05$, $\alpha = 0.05$, $\delta = 3.5$, $\kappa = 0.01$, $\nu = 40$. The following prior distributions are assumed:

$$\omega \sim N(0, 1), \quad \beta \sim Beta(8, 1), \quad \theta \sim N(0, 1), \quad \alpha \sim N(0, 1)$$

$$\delta^{-2} \sim Gamma(4, 75), \quad \kappa \sim Beta(2, 100), \quad \nu \sim Gamma(3, 0.1)$$

These values of parameters and prior distributions echo past literature as well as the author’s prior belief (Nakajima (2008)). For $\omega$, $\theta$ and $\alpha$, we choose the relatively non-informative standard Normal as the priors. $\beta$ is specified to be between 0 and 1 in the model and is therefore assigned a Beta distribution. To reflect the high persistence in volatility, we construct this Beta distribution to have a density concentrated towards 1.

As for the jump and heavy-tail components, we use a Gamma distribution for $\delta$; we use a Beta distribution whose density is concentrated towards 0 for $\kappa$, which represents the success rate in a Bernoulli distribution\(^1\). $\nu$ is the degree of freedom for the Student-t distribution. We choose a Gamma distribution with mean of 30 and a big standard deviation of 17 to ensure that it is spread out enough for all levels of heavy-tailness in the data.

4.2 Estimation Results

We estimate the parameters following the MCMC algorithm outlined in Section 3.2. The proposal distributions used for the Metropolis-Hastings in estimating $\vartheta$, $\delta$ and $\nu$ are updated in each MCMC iteration based on current values of the rest of the parameters\(^2\). We draw 1,000 posterior samples after discarding the first 30,000 out of 80,000 samples and thinning the rest 1 out of 50 (to ensure low auto-correlations).

Figure (1) shows the sample traceplots for each parameter and Table (1) summarizes MCMC diagnostics. Sample paths look stable and suggest good mixing of the chains. Reasonable acceptance rates and Geweke statistics further confirm the effectiveness of the method used.

\(^1\)A slightly different set of priors is also used for the jump component, but makes no significant difference on posterior inference.

\(^2\)Note that estimating parameters with fixed proposal distributions is also carried out; results are not materially different.
Figure 1: Traceplots for 1000 Posterior Samples (Simulated Data)

Table 1: MCMC Diagnostics for 1000 Posterior Samples (Simulated Data)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Acceptance rate</th>
<th>Effective size</th>
<th>Lag 1 auto-correl</th>
<th>Geweke diagnostic</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\omega$</td>
<td>22%</td>
<td>670</td>
<td>0.036</td>
<td>-0.80</td>
</tr>
<tr>
<td>$\beta$</td>
<td>22%</td>
<td>1000</td>
<td>0.006</td>
<td>-0.05</td>
</tr>
<tr>
<td>$\theta$</td>
<td>22%</td>
<td>1000</td>
<td>0.028</td>
<td>-0.81</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>22%</td>
<td>1000</td>
<td>-0.040</td>
<td>0.46</td>
</tr>
<tr>
<td>$\delta$</td>
<td>35%</td>
<td>1000</td>
<td>0.037</td>
<td>0.99</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>100%</td>
<td>446</td>
<td>0.051</td>
<td>-0.06</td>
</tr>
<tr>
<td>$\nu$</td>
<td>32%</td>
<td>39</td>
<td>0.924</td>
<td>0.05</td>
</tr>
</tbody>
</table>
Table (2) gives the estimates for posterior means, standard deviations and the 95% intervals. The estimated posterior means are close to the true values, and the true values are all contained in their corresponding 95% intervals. Looking at the plots in Figure (2), posterior densities show smooth patterns and all shift from the priors toward the true values.

Table 2: Estimation results with simulated data for EGARCH\(J_t\) model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>True</th>
<th>Mean</th>
<th>Stdev.</th>
<th>0.25%</th>
<th>97.5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\omega)</td>
<td>-0.02</td>
<td>-0.018</td>
<td>0.005</td>
<td>(-0.030, -0.010)</td>
<td></td>
</tr>
<tr>
<td>(\beta)</td>
<td>0.95</td>
<td>0.950</td>
<td>0.010</td>
<td>(0.928, 0.967)</td>
<td></td>
</tr>
<tr>
<td>(\theta)</td>
<td>-0.05</td>
<td>-0.059</td>
<td>0.013</td>
<td>(-0.084, -0.034)</td>
<td></td>
</tr>
<tr>
<td>(\alpha)</td>
<td>0.15</td>
<td>0.170</td>
<td>0.020</td>
<td>(0.132, 0.213)</td>
<td></td>
</tr>
<tr>
<td>(\delta)</td>
<td>3.50</td>
<td>4.041</td>
<td>0.783</td>
<td>(2.880, 5.986)</td>
<td></td>
</tr>
<tr>
<td>(\kappa)</td>
<td>0.01</td>
<td>0.007</td>
<td>0.003</td>
<td>(0.002, 0.013)</td>
<td></td>
</tr>
<tr>
<td>(\nu)</td>
<td>40.00</td>
<td>28.86</td>
<td>11.13</td>
<td>(14.09, 55.52)</td>
<td></td>
</tr>
</tbody>
</table>

Figure 2: Posterior Densities (Solid Line) vs. Prior Densities (Dotted Line)
5 Data

Emerging Market equities have become increasingly popular among investors as an ideal device for diversification in the past decade. While U.S. equity volatility has been extensively studied, the behavior of volatility in Emerging Markets is just as complicated and provides valuable information for money managers when optimizing portfolios and hedging risks. We estimate the EGARCHJt for weekly international equity returns, obtained from Russell Investment (Egger (2010))\(^3\). Our data comprises weekly returns on the overall Russell 3000 Index, the overall EM Growth Index and thirteen EM countries/clusters\(^4\), from July 3rd, 1996 to May 20th, 2009. The sample size is 673 for each series\(^5\).

Table (3) summarizes the descriptive statistics of the data. Most series have mean returns very close to zero. Among these countries/clusters, EMEA has the lowest standard deviation and Turkey the highest. They all exhibit kurtosis larger than that of a Normal distribution, with Thailand having the highest kurtosis of 19.22. Figure (3) plots the time-series returns for selected countries. Consistent with empirical and theoretical suggestions of “volatility clustering”, changes in equity returns show high persistence.

<table>
<thead>
<tr>
<th>Mean</th>
<th>Stdev</th>
<th>Skewness</th>
<th>Kurtosis</th>
<th>Min.</th>
<th>Max.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0010</td>
<td>0.031</td>
<td>-0.419</td>
<td>7.614</td>
<td>-0.192</td>
<td>0.164</td>
</tr>
<tr>
<td>0.0006</td>
<td>0.032</td>
<td>-0.388</td>
<td>7.846</td>
<td>-0.174</td>
<td>0.199</td>
</tr>
<tr>
<td>0.0027</td>
<td>0.047</td>
<td>-0.469</td>
<td>6.066</td>
<td>-0.266</td>
<td>0.207</td>
</tr>
<tr>
<td>0.0012</td>
<td>0.028</td>
<td>-0.159</td>
<td>6.076</td>
<td>-0.160</td>
<td>0.142</td>
</tr>
<tr>
<td>0.0024</td>
<td>0.055</td>
<td>0.146</td>
<td>9.602</td>
<td>-0.316</td>
<td>0.332</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.073</td>
<td>0.314</td>
<td>9.960</td>
<td>-0.334</td>
<td>0.433</td>
</tr>
<tr>
<td>0.0028</td>
<td>0.047</td>
<td>-0.311</td>
<td>5.569</td>
<td>-0.241</td>
<td>0.212</td>
</tr>
<tr>
<td>0.0020</td>
<td>0.037</td>
<td>-0.359</td>
<td>8.721</td>
<td>-0.240</td>
<td>0.226</td>
</tr>
<tr>
<td>-0.0001</td>
<td>0.049</td>
<td>0.635</td>
<td>16.619</td>
<td>-0.310</td>
<td>0.407</td>
</tr>
<tr>
<td>-0.0008</td>
<td>0.056</td>
<td>1.707</td>
<td>19.220</td>
<td>-0.194</td>
<td>0.551</td>
</tr>
<tr>
<td>0.0037</td>
<td>0.074</td>
<td>0.416</td>
<td>9.951</td>
<td>-0.314</td>
<td>0.587</td>
</tr>
<tr>
<td>0.0016</td>
<td>0.040</td>
<td>0.107</td>
<td>14.157</td>
<td>-0.236</td>
<td>0.326</td>
</tr>
<tr>
<td>0.0020</td>
<td>0.028</td>
<td>-0.951</td>
<td>8.410</td>
<td>-0.163</td>
<td>0.130</td>
</tr>
<tr>
<td>-0.0007</td>
<td>0.047</td>
<td>0.313</td>
<td>7.358</td>
<td>-0.222</td>
<td>0.279</td>
</tr>
<tr>
<td>0.0010</td>
<td>0.031</td>
<td>-0.316</td>
<td>9.276</td>
<td>-0.170</td>
<td>0.220</td>
</tr>
</tbody>
</table>

\(^3\)Clean country level weekly data is provided by Professor Daniel Egger from the Center of Quantitative Modeling at Duke University.

\(^4\)Countries in the overall EM Growth Index that contain fewer than 15 equities at any time during the sample period are grouped into three regional clusters: Asia, S.America (South America), and EMEA (Europe, Middle East, and Africa). See Appendix C for a complete list of countries and cluster components.

\(^5\)Note that we standardize our data when fitting the EGARCHJt model.
Figure 3: Return series for selected countries (7/3/1996 - 5/20/2009)

Russell 3000

EM Growth

China

Thailand

Brazil

Turkey
6 Results

In this section, we apply the EGARCHJt model to weekly international equity returns. Prior specifications are reported in Appendix D. We first fit the model to the overall Russell 3000 Index and the overall EM Growth Index, with a sample size of 673 for each from 1996 to 2009. In this analysis, “Russell 3000” represents the U.S. equity market and “EM Growth” represents the Emerging Markets (EM). In addition, we fit the model to thirteen countries/clusters in EM, covering Asia, Europe, Africa, the Middle East, and South America.

Based on the posterior estimates of the parameters, we compare volatility characteristics in those countries and regions. In Section 6.1, we discuss the dependence of current period’s volatility on the previous period’s volatility and previous period’s return. In Section 6.2, we examine the overall tail behavior of the returns. In each subsection, we first compare parameter values for the Russell 3000 and EM Growth indices to illustrate the difference between developed and developing markets. We then focus on individual countries/clusters in EM to highlight any existing features of volatility behavior.

6.1 Persistence and Leverage Effects

Table (4) reports the posterior parameter estimates for $\beta$, $\alpha$ and $\theta$. As mentioned in the model specification in Section 2, parameter $\beta$ shows how dependent the current period’s log conditional variance is on the previous period’s log conditional variance. In other words, the closer the value of $\beta$ is to 1, the more “persistent” volatility is over time in that market. In addition to previous volatility, past return also contributes to current volatility. Parameter $\alpha$ represents the magnitude of such impact on current volatility from unexpected shock in previous equity return.

Comparing the two overall indices first, the posterior of $\beta$ is higher in the U.S. (with a mean of 0.956) than in EM (with a mean of 0.909), while posterior of $\alpha$ is lower in the U.S (with a mean of 0.158) than in EM (with a mean of 0.296). This result suggests that current volatility in the U.S. is more dependent on past volatility and is less effected by unexpected shock in past return compared to EM. It is reasonable to observe higher volatility persistence in more mature markets, as assets tend to move in one direction for longer than they do in younger markets. At the same time, people investing in the U.S. watch individual stocks in the index much more closely and thus react less to return shocks, while an unexpected change in return may cause more panic in EM.

Looking at the individual countries/clusters, posterior draws for $\beta$ are mostly greater than 0.8, implying existence of “volatility clustering” in EM. Malaysia, China and Thailand exhibit exceedingly high volatility persistence compared to their EM peers with $\beta$ means of 0.975, 0.959 and 0.956 respectively, surpassing the level in the U.S. This might be explained by the fact that the Malaysian and Thai markets are highly volatile over time, and the Chinese market is closely tracked by investors. The South America cluster has the lowest persistence with a mean of 0.853. But at the same
time, it has the largest “magnitude effect” with a mean of $\alpha$ at 0.358. Note that $\beta$ and $\alpha$ are often analyzed together to provide information for volatility prediction based on the previous period’s volatility and return.

The parameter $\theta$ indicates “leverage effects”, the asymmetric impact on current volatility from positive and negative past return. They are estimated to be negative for both Russell 3000 and EM Growth, and the 95% posterior intervals do not contain zero, which confirms the existence of leverage effects in developed and developing markets. The magnitude of $\theta$ is slightly larger in the U.S. (with a mean of -0.145) than in EM (with a mean of -0.132).

For individual countries/clusters, the values of $\theta$ are estimated to be negative in most cases, with the exception of Chile having a 95% interval of (-0.079, 0.022). This result suggests that volatility in the Chile market sometimes responds more to a positive past shock than to a negative one, which can be useful information for investors when they construct portfolios. Besides Chile, China has relatively small leverage effects with a mean of $\theta$ at -0.052 compared to its EM peers.

Table 4: Summary Results for $\beta$, $\alpha$ and $\theta$

<table>
<thead>
<tr>
<th></th>
<th>$\beta$</th>
<th></th>
<th>$\alpha$</th>
<th></th>
<th>$\theta$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5%</td>
<td>Mean</td>
<td>97.5%</td>
<td>2.5%</td>
<td>Mean</td>
<td>97.5%</td>
</tr>
<tr>
<td>Russell 3000</td>
<td>0.923</td>
<td>0.956</td>
<td>0.979</td>
<td>0.084</td>
<td>0.158</td>
<td>0.255</td>
</tr>
<tr>
<td>EM Growth</td>
<td>0.849</td>
<td>0.909</td>
<td>0.955</td>
<td>0.199</td>
<td>0.296</td>
<td>0.406</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.897</td>
<td>0.946</td>
<td>0.986</td>
<td>0.158</td>
<td>0.262</td>
<td>0.373</td>
</tr>
<tr>
<td>China</td>
<td>0.819</td>
<td>0.911</td>
<td>0.975</td>
<td>0.140</td>
<td>0.262</td>
<td>0.392</td>
</tr>
<tr>
<td>Chile</td>
<td>0.923</td>
<td>0.959</td>
<td>0.986</td>
<td>0.184</td>
<td>0.275</td>
<td>0.380</td>
</tr>
<tr>
<td>Indonesia</td>
<td>0.839</td>
<td>0.906</td>
<td>0.953</td>
<td>0.253</td>
<td>0.353</td>
<td>0.468</td>
</tr>
<tr>
<td>India</td>
<td>0.813</td>
<td>0.907</td>
<td>0.971</td>
<td>0.130</td>
<td>0.222</td>
<td>0.328</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.895</td>
<td>0.947</td>
<td>0.983</td>
<td>0.079</td>
<td>0.155</td>
<td>0.255</td>
</tr>
<tr>
<td>Malaysia</td>
<td>0.948</td>
<td>0.975</td>
<td>0.992</td>
<td>0.123</td>
<td>0.209</td>
<td>0.313</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.884</td>
<td>0.956</td>
<td>0.991</td>
<td>0.097</td>
<td>0.226</td>
<td>0.393</td>
</tr>
<tr>
<td>Turkey</td>
<td>0.817</td>
<td>0.909</td>
<td>0.974</td>
<td>0.140</td>
<td>0.253</td>
<td>0.386</td>
</tr>
<tr>
<td>S.Africa</td>
<td>0.838</td>
<td>0.913</td>
<td>0.967</td>
<td>0.014</td>
<td>0.199</td>
<td>0.312</td>
</tr>
<tr>
<td>EMEA</td>
<td>0.881</td>
<td>0.936</td>
<td>0.974</td>
<td>0.187</td>
<td>0.288</td>
<td>0.400</td>
</tr>
<tr>
<td>Asia</td>
<td>0.864</td>
<td>0.931</td>
<td>0.975</td>
<td>0.146</td>
<td>0.261</td>
<td>0.383</td>
</tr>
<tr>
<td>S.America</td>
<td>0.748</td>
<td>0.853</td>
<td>0.935</td>
<td>0.214</td>
<td>0.358</td>
<td>0.505</td>
</tr>
</tbody>
</table>
6.2 Tail Behavior: Jumps and Student-t Distribution

We then investigate the behavior of excess returns implied by the jumps and heavy-tails specified in the model. Table (5) reports posterior estimates for $\kappa$, $\delta$ and $\nu$, the parameters that characterize jump probability, jump size and heavy-tailness in terms of the degrees of freedom for a Student-t distribution respectively. The jump component identifies large discrete variations in returns that are not captured by the Student-t distribution, while the Student-t accounts for the remaining behavior in the tail. Since the jump and heavy-tail components are similar in capturing excess returns in the distribution, they are interpreted together to provide an overall description of the tail. Due to the sample size and frequency of our data, the tail behaviors are not as distinct as they would have been had we used daily data, resulting in the jump size parameter $\delta$ being sensitive to its prior as well as the degrees of freedom parameter $\nu$ having large posterior standard deviation. Therefore, we examine these parameters on a relative basis across the markets.

As can be seen in Table (5), U.S. and EM have similar behaviors in terms of jumps. Comparing the posterior means of $\kappa$ and $\delta$, EM contains jumps with a slightly higher probability of 0.9% than the 0.8% in the U.S., while the average jump size in EM, 3.38, is slightly smaller compared to the 3.49 in the U.S. On the other hand, the U.S. equity returns have lighter tails than EM, implied by the estimates of $\nu$. This result is expected as the U.S. market is more mature and sophisticated than EM, and functions in a manner that is closer to the “efficient market” assumption in which asset returns follow a Normal distribution.

Examining the parameter values for the EM countries/clusters, we find that Turkey has the lowest probability of large excess returns, indicated by the posterior mean of its $\kappa$ value, 0.3%. The remaining parts of EM have equal or higher jump probabilities than the U.S., with posterior means of $\kappa$ ranging from 0.8% in China and Malaysia to 1.6% in Thailand. Based on the 95% posterior intervals of $\kappa$, most countries/clusters have jumps contribute to no more than 3.5% of the total returns.

All the EM countries/clusters have lower degrees of freedom for the Student-t distribution than the U.S., suggested by the posterior values of $\nu$. Amongst all, Indonesia exhibits heaviest tail in its return distribution with a mean of $\nu$ at 6.78. Its neighboring country, Malaysia, has similar posterior $\nu$ values with a mean of 7.11. The Asia Cluster\(^6\) that comprises returns from Philippines, Vietnam and Laos also has a thick tail with a mean of $\nu$ at 7.13. These findings provide useful insights into the equity behavior in the Southeast Asia region.

Since the jump component and Student-t distribution are both devices to account for excess variations in the equity return distribution, it is meaningful to understand the relationship between them. Turkey and Thailand are good examples here to illustrate the interactions between the parameters. Although not many jumps are identified in the Turkish market based on our specification, the jumps captured by the model are estimated to have the largest size among all EM countries/clusters,

---

\(^6\)Over 90% of the Asia Cluster is Filipino stocks.
with a $\delta$ mean of 3.73. In other words, there are not many incidences of excess returns in Turkey, but they tend to be relatively big when there are. At the same time, Turkey shows low degrees of freedom with a mean of $\nu$ at 8.00, implying significant weights of the distribution in the tails. This finding suggests that most big variations in Turkish equities are captured in the overall behavior in the Student-t distribution, with a small number of exceptionally large excess returns further away from the tails. Thailand, on the other hand, has much higher jump probability than Turkey, but its $\nu$ (with a mean of 21.92) is estimated to be much higher than that of Turkey as well as the other countries in Southeast Asia. This suggests that Thailand’s large excess returns are mostly captured by the jumps instead of the Student-t distribution.

We can also associate the estimated results of the jump flag $\gamma_t$ with events in the financial world. Despite the fact that our weekly equity data contains limited information about the tails, the model is still able to recognize some big changes in the return series. For example, the model identifies jumps in the Thai market on the week of 01/28/1998, amidst the Asian crisis, for about 85% of the time. The model also identifies jumps in the U.S. and Mexican markets on the week of 10/01/2008, two weeks after Lehman Brothers filed for bankruptcy, for about 80% of the time.

Table 5: Summary Results for $\kappa$, $\delta$ and $\nu$

<table>
<thead>
<tr>
<th></th>
<th>$\kappa$</th>
<th></th>
<th></th>
<th>$\delta$</th>
<th></th>
<th></th>
<th>$\nu$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.5%</td>
<td>Mean</td>
<td>97.5%</td>
<td>2.5%</td>
<td>Mean</td>
<td>97.5%</td>
<td>2.5%</td>
<td>Mean</td>
<td>97.5%</td>
</tr>
<tr>
<td>Russell 3000</td>
<td>0.001</td>
<td>0.008</td>
<td>0.019</td>
<td>2.32</td>
<td>3.49</td>
<td>5.35</td>
<td>15.24</td>
<td>37.01</td>
<td>81.60</td>
</tr>
<tr>
<td>EM Growth</td>
<td>0.001</td>
<td>0.009</td>
<td>0.024</td>
<td>2.26</td>
<td>3.38</td>
<td>5.18</td>
<td>9.64</td>
<td>24.33</td>
<td>54.20</td>
</tr>
<tr>
<td>Brazil</td>
<td>0.001</td>
<td>0.009</td>
<td>0.025</td>
<td>2.27</td>
<td>3.37</td>
<td>5.43</td>
<td>8.10</td>
<td>21.50</td>
<td>52.08</td>
</tr>
<tr>
<td>Chile</td>
<td>0.001</td>
<td>0.010</td>
<td>0.027</td>
<td>2.28</td>
<td>3.39</td>
<td>5.20</td>
<td>7.66</td>
<td>23.62</td>
<td>62.67</td>
</tr>
<tr>
<td>China</td>
<td>0.001</td>
<td>0.008</td>
<td>0.022</td>
<td>2.31</td>
<td>3.43</td>
<td>5.22</td>
<td>6.63</td>
<td>14.81</td>
<td>35.40</td>
</tr>
<tr>
<td>Indonesia</td>
<td>0.003</td>
<td>0.012</td>
<td>0.029</td>
<td>2.45</td>
<td>3.55</td>
<td>5.51</td>
<td>3.58</td>
<td>6.78</td>
<td>17.69</td>
</tr>
<tr>
<td>India</td>
<td>0.003</td>
<td>0.015</td>
<td>0.034</td>
<td>2.29</td>
<td>3.29</td>
<td>4.88</td>
<td>8.39</td>
<td>24.35</td>
<td>59.04</td>
</tr>
<tr>
<td>Mexico</td>
<td>0.003</td>
<td>0.014</td>
<td>0.031</td>
<td>2.40</td>
<td>3.38</td>
<td>5.07</td>
<td>6.50</td>
<td>17.74</td>
<td>41.32</td>
</tr>
<tr>
<td>Malaysia</td>
<td>0.001</td>
<td>0.008</td>
<td>0.021</td>
<td>2.28</td>
<td>3.44</td>
<td>5.51</td>
<td>3.96</td>
<td>7.11</td>
<td>15.84</td>
</tr>
<tr>
<td>Thailand</td>
<td>0.004</td>
<td>0.016</td>
<td>0.035</td>
<td>2.50</td>
<td>3.63</td>
<td>5.43</td>
<td>7.06</td>
<td>21.92</td>
<td>55.61</td>
</tr>
<tr>
<td>Turkey</td>
<td>0.000</td>
<td>0.003</td>
<td>0.007</td>
<td>2.41</td>
<td>3.73</td>
<td>6.17</td>
<td>4.54</td>
<td>8.00</td>
<td>17.18</td>
</tr>
<tr>
<td>S.Africa</td>
<td>0.003</td>
<td>0.014</td>
<td>0.031</td>
<td>2.43</td>
<td>3.47</td>
<td>5.17</td>
<td>6.55</td>
<td>17.53</td>
<td>48.76</td>
</tr>
<tr>
<td>EMEA</td>
<td>0.001</td>
<td>0.009</td>
<td>0.025</td>
<td>2.27</td>
<td>3.35</td>
<td>5.33</td>
<td>5.76</td>
<td>15.47</td>
<td>41.98</td>
</tr>
<tr>
<td>Asia</td>
<td>0.001</td>
<td>0.010</td>
<td>0.028</td>
<td>2.17</td>
<td>3.29</td>
<td>5.09</td>
<td>3.99</td>
<td>7.13</td>
<td>17.43</td>
</tr>
<tr>
<td>S.America</td>
<td>0.002</td>
<td>0.012</td>
<td>0.032</td>
<td>2.18</td>
<td>3.25</td>
<td>4.98</td>
<td>6.96</td>
<td>18.96</td>
<td>48.89</td>
</tr>
</tbody>
</table>
7 Conclusion

In this paper, we carry out Bayesian inference on the EGARCHJt model through MCMC, and apply the model to weekly equity data in both U.S. and Emerging Markets. The Metropolis-Hastings and Gibbs sampling algorithms are shown to be effective in estimating the EGARCHJt parameters. Based on the posterior distributions of these parameters, we examine the way past volatility and return contribute to current period’s volatility and investigate the tail behaviors in the returns. Results suggest that volatility in U.S. displays more persistence than in EM. The U.S. market also has bigger leverage effects, fewer jumps and a return distribution closer to Normal. Within the EM, Malaysia and China exhibit the highest volatility persistence; Thailand and India have the highest jump probability, and Indonesia and Malaysia are the heaviest in the Student-t tails.

Our analysis is performed with a relatively small sample size and data collected on a weekly basis. As a result, the tail behaviors are not as distinct as they would have been had we used daily data. It would be of interest to carry out the same exercise using data collected on a more frequent rate. With more information regarding the distribution tails, research can be done to show the spread of financial crises in one market to another as well as examine the existence of time lags of financial shocks based on jump identifications.
A Derivation of the Student-t Distribution

We specify $\lambda_t^{-1}$ to be $\text{Gamma}(\nu/2, \nu/2)$ and therefore

$$\nu \lambda_t^{-1} \sim \text{Gamma}(\nu/2, 1/2)$$

which is equivalent to a Chi-square distribution with $\nu$ degrees of freedom. Since $\epsilon_t$ is standard normal, we know that

$$\sqrt{\lambda_t} \epsilon_t = \frac{\epsilon_t}{\sqrt{\nu \lambda_t^{-1}/\nu}}$$

follows a Student-t distribution with $\nu$ degrees of freedom.
B Empirical Estimation Procedure for $\tilde{\vartheta}$

Let $\Psi = (\delta, \gamma, \lambda)$ for notational purposes. A single iteration to obtain the empirical value of $\tilde{\vartheta}$ is shown below:

1) Find $\hat{\omega}^{(j)}$ that maximizes

$$
\log L(\omega | \hat{\beta}^{(j-1)}, \hat{\theta}^{(j-1)}, \hat{\alpha}^{(j-1)}, \Psi, y) = \log \pi(\omega) + \sum_{t=1}^{n} \frac{-1}{2} \log(\gamma_t \delta^2 + \lambda_t \hat{\sigma}^2_t) - \frac{y_t^2}{2(\gamma_t \delta^2 + \lambda_t \hat{\sigma}^2_t)}
$$

where $\hat{\beta}^{(j-1)}, \hat{\theta}^{(j-1)}, \hat{\alpha}^{(j-1)}$ are values from the previous step, and

$$
\hat{\sigma}^2_t = \hat{\sigma}^2_t(\omega) = \exp \left\{ \omega + \hat{\beta}^{(j-1)} \log \hat{\sigma}^2_{t-1}(\omega) + \hat{\theta}^{(j-1)} \left( \frac{y_{t-1}}{\hat{\sigma}_{t-1}} \right) + \hat{\alpha}^{(j-1)} \left( \left| \frac{y_{t-1}}{\hat{\sigma}_{t-1}} \right| - \zeta \right) \right\}
$$

2) Find $\hat{\beta}^{(j)}$ that maximizes

$$
\log L(\beta | \hat{\omega}^{(j)}, \hat{\theta}^{(j-1)}, \hat{\alpha}^{(j-1)}, \Psi, y) = \log \pi(\beta) + \sum_{t=1}^{n} \frac{-1}{2} \log(\gamma_t \delta^2 + \lambda_t \hat{\sigma}^2_t) - \frac{y_t^2}{2(\gamma_t \delta^2 + \lambda_t \hat{\sigma}^2_t)}
$$

where

$$
\hat{\sigma}^2_t = \hat{\sigma}^2_t(\beta) = \exp \left\{ \hat{\omega}^{(j)} + \beta \log \hat{\sigma}^2_{t-1}(\beta) + \hat{\theta}^{(j-1)} \left( \frac{y_{t-1}}{\hat{\sigma}_{t-1}} \right) + \hat{\alpha}^{(j-1)} \left( \left| \frac{y_{t-1}}{\hat{\sigma}_{t-1}} \right| - \zeta \right) \right\}
$$

3) Find $\hat{\theta}^{(j)}$ that maximizes

$$
\log L(\theta | \hat{\omega}^{(j)}, \hat{\beta}^{(j)}, \hat{\alpha}^{(j-1)}, \Psi, y) = \log \pi(\theta) + \sum_{t=1}^{n} \frac{-1}{2} \log(\gamma_t \delta^2 + \lambda_t \hat{\sigma}^2_t) - \frac{y_t^2}{2(\gamma_t \delta^2 + \lambda_t \hat{\sigma}^2_t)}
$$

where

$$
\hat{\sigma}^2_t = \hat{\sigma}^2_t(\theta) = \exp \left\{ \hat{\omega}^{(j)} + \hat{\beta}^{(j)} \log \hat{\sigma}^2_{t-1}(\theta) + \theta \left( \frac{y_{t-1}}{\hat{\sigma}_{t-1}} \right) + \hat{\alpha}^{(j-1)} \left( \left| \frac{y_{t-1}}{\hat{\sigma}_{t-1}} \right| - \zeta \right) \right\}
$$

4) Find $\hat{\alpha}^{(j)}$ that maximizes

$$
\log L(\alpha | \hat{\omega}^{(j)}, \hat{\beta}^{(j)}, \hat{\theta}^{(j)}, \Psi, y) = \log \pi(\alpha) + \sum_{t=1}^{n} \frac{-1}{2} \log(\gamma_t \delta^2 + \lambda_t \hat{\sigma}^2_t) - \frac{y_t^2}{2(\gamma_t \delta^2 + \lambda_t \hat{\sigma}^2_t)}
$$

where

$$
\hat{\sigma}^2_t = \hat{\sigma}^2_t(\alpha) = \exp \left\{ \hat{\omega}^{(j)} + \hat{\beta}^{(j)} \log \hat{\sigma}^2_{t-1}(\alpha) + \hat{\theta}^{(j)} \left( \frac{y_{t-1}}{\hat{\sigma}_{t-1}} \right) + \alpha \left( \left| \frac{y_{t-1}}{\hat{\sigma}_{t-1}} \right| - \zeta \right) \right\}
$$

We repeat 1) through 4) for $J$ times ($J = 5$ will usually suffice) and set

$$
\tilde{\vartheta} = (\tilde{\omega}, \tilde{\beta}, \tilde{\theta}, \tilde{\alpha}) = (\hat{\omega}^{(J)}, \hat{\beta}^{(J)}, \hat{\theta}^{(J)}, \hat{\alpha}^{(J)})
$$

Note that $\tilde{\vartheta}$ does not give the global maximum of $\pi(\vartheta | \delta, \gamma, \lambda, y)$, but is good enough for the purpose of the proposal distribution.
## C Complete List of Countries and Clusters

<table>
<thead>
<tr>
<th>Countries</th>
<th>EMEA Cluster</th>
<th>S.America Cluster</th>
<th>Asia Cluster</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brazil</td>
<td>Morocco</td>
<td></td>
<td>Philippines</td>
</tr>
<tr>
<td>Chile</td>
<td>Oman</td>
<td>Argentina</td>
<td>Vietnam</td>
</tr>
<tr>
<td>China</td>
<td>Mauritius</td>
<td>Venezuela</td>
<td>Laos</td>
</tr>
<tr>
<td>Indonesia</td>
<td>Pakistan</td>
<td>Peru</td>
<td></td>
</tr>
<tr>
<td>India</td>
<td>Qatar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mexico</td>
<td>Poland</td>
<td>Panama</td>
<td></td>
</tr>
<tr>
<td>Malaysia</td>
<td>Portugal</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Thailand</td>
<td>Romania</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Turkey</td>
<td>Russia</td>
<td></td>
<td></td>
</tr>
<tr>
<td>South Africa</td>
<td>Slovenia</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Slovakia</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tunisia</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Ukraine</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Latvia</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Libya</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Greece</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Egypt</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Czech Republic</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Cyprus</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Botswana</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bahrain</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Bulgaria</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Croatia</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Hungary</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kazakhstan</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Kuwait</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Jordan</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>United Arab Emirates</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Israel</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
D  Prior Specifications for EGARCHJt

The following prior distributions are assumed:

\[ \omega \sim \text{N}(0, 1), \quad \beta \sim \text{Beta}(8, 1), \quad \theta \sim \text{N}(0, 1), \quad \alpha \sim \text{N}(0, 1) \]

\[ \delta^{-2} \sim \text{Gamma}(5, 60), \quad \kappa \sim \text{Beta}(2, 100), \quad \nu \sim \text{Gamma}(3, 0.1) \]
References


