LAST NAME (Please Print): KEY

FIRST NAME (Please Print): ____________________________________________

HONOR PLEDGE (Please Sign): __________________________________________

Statistics 101

Homework 4

You are allowed to discuss problems with other students, but the final answers must be your own work.

For all problems that require calculation, YOU MUST ATTACH SEPARATE PAGES, NEATLY WRITTEN, THAT SHOW YOUR WORK.

Please mark your answer in the space provided. As a general rule, each blank counts for one point. If necessary work is not shown, or if that work is substantially wrong, then you will not get credit even if the answer is correct. (The obvious purpose of this seemingly draconian policy is to prevent people from mindlessly copying each other’s answers.)

Report all numerical answers to at least two correct decimal places.

Upload to Gradescope by 3:30 p.m. on Feb. 15.
1. Use the data on women in the labor force that is available at:
   http://lib.stat.cmu.edu/DASL/Datafiles/LaborForce.html

We want to test, at the .025 level, whether there is evidence of increasing numbers of women working outside the home.

One approach to doing this is to compare the average proportion in 1968 to the average proportion in 1972. Another approach is to take the difference in proportions over time for each city, and then test whether the average difference is greater than zero.

For the first approach, write the null hypothesis in symbols.

\[ H_0 : \mu_{1972} - \mu_{1968} \leq 0. \]

This question is asking for a difference of averages. The fact that the things being averaged are proportions misleads some people into thinking that they should use a hypothesis test about proportions, but that is a mistake.

In words, the null is The average proportion in 1972 minus the average proportion in 1968 is less than or equal to zero.

Note: You could subtract 1972 from 1968, and if you did things consistently through the rest of the problem, you would get the right answer, you’d just be rejecting for values in the lower tail rather than the upper tail.

Write the alternative in words.

This is what we want to show. Alternative: The average proportion in 1972 minus the average proportion in 1968 is greater than zero.

1.54 Suppose that the population sd equals the sample sd. (We make this assumption so that one of the methods we have covered will apply.) What is the value of your test statistic?

Nearly all tests have the form (point estimate - null value)/standard error. Here the point estimate of the difference is the 1972 average minus the 1968 average, or \(.5268 - .4932 = .0337\). The null value for the difference is 0. And the standard error is, according to our formula, \(\sqrt{SD_1^2/n_1 + SD_2^2/n_2} = \sqrt{.0689^2/19 + .0662^2/19} = .0219\). So \((.0337 - 0)/.0219 = 1.54\).

What distribution should your test statistic follow when the null hypothesis is true? (Under our assumption that the sd is known.)
What is your significance probability?

From our z-table, the nearest value to 1.54 is 1.55. \( P[Z > 1.55] = .06055 \). (If you use a more exact table, you get .0622.

What conclusion do you reach?

We cannot reject the null hypothesis that there is no difference at the .025 level, since the P-value, .061, is larger than .025.

Set a 95% one-sided lower confidence bound on the difference in the average proportion of women in the workforce in 1972 and the proportion in 1968 (the later year minus the earlier).

The general formula for a one-sided interval is \( pe + se \times cv \). The point estimate is \( pe = .0337 \), the standard error is .02192, and the critical value is \( cv = -1.65 \) (this is a lower bound, so we use the negative side of the standard normal distribution, and we find the value that has 95% of the area under the curve and to the right.

For the second approach, write the null hypothesis in words.

Null: The average difference in proportions is less than or equal to zero (i.e., the average proportion of women in the workforce has not increased).

Write the alternative in symbols.

\[ H_A : \mu_{1972} - \mu_{1968} > 0. \]

The average difference in proportions is positive (i.e., the average proportion of women in the workforce has increased).

What is the value of your test statistic?

We find the pairwise differences for each of the 19 cities. The average of these difference is the same as the previous difference of averages, or 0.03368. The standard deviation of these 19 differences is \( SD_d = .05974 \). The test statistic is \( (pe - \text{null value})/se = (0.03368 - 0)/(.05974/\sqrt{19}) = 2.4577 \).
What distribution does the test statistic follow when the null is true?

between .01 and .025 What is your significance probability?

Using the t-table, in the row with df=18, the significance probability is between 1% and 2.5%.

What conclusion do you reach?

At the .025 level, we can reject the null hypothesis. There is evidence that the average proportion of women in the workforce has increased.

.00997 Set a 95% one-sided lower confidence bound on the average difference in the proportion of women in the workforce between 1972 and 1968 (the later year minus the earlier).

As before: \(pe + se \times cv\). The \(pe\) is .03368, the \(se\) is now \(0.05974/\sqrt{19}\), and the \(cv\) is -1.73, from the t-table.

Why is the second approach more powerful than the first?

The second approach controls for variation between cities. The first approach does not. Since there is large variation among cities in the proportion of women in the work force, it masks the fact that most cities made small improvements during this time period.

2. You want set a two-sided 90% confidence interval on the median verbal SAT score of Duke students. Your initial sample of 50 students had a median of 685, and 20 bootstrap samples had the following medians:

\[
650, 700, 710, 720, 780, 800, 600, 680, 670, 790, 680, 590, 600, 630, 650, 720, 740, 760, 670, 690
\]

What is your best interval? Hint: Recall the rules for finding interquartile ranges.

Since this is a CI on a median, we must use the bootstrap, and the best bootstrap is the pivot bootstrap.
First, order the numbers from least to largest. You find:

\[
590, 600, 600, 630, 650, 650, 670, 670, 680, 680, \\
690, 700, 710, 720, 720, 740, 760, 780, 790, 800
\]

The 95th percentile is any number between 790 and 800; the IQR rule says to take 795. Similarly, the 5th percentile is 595. Thus the pivot bounds are \(2 \times 685 - 795\) and \(2 \times 685 - 595\).

\[
L = 575 \quad U = 775
\]

3. A Fox News reporter claims that at least 10% more women than men vote for the more handsome candidate. You want to prove him wrong. You draw a random sample of 100 men and 150 women, and ask them whether they would vote for Orlando Bloom if he ran against Newt Gingrich. (Assume Bloom is more handsome than Gingrich.) You find that 80 men would vote for Bloom, and so would 125 women.

In symbols, what is the null hypothesis? \(H_0 : p_w - p_m \geq 0.1\)

Note to TA: Accept any mathematically equivalent statement of the null hypothesis.

-1.33 What is the value of your test statistic?

\[
ts = \frac{\hat{p}_w - \hat{p}_m - 0.1}{\sqrt{\hat{p}_w(1 - \hat{p}_w)/n_w + \hat{p}_m(1 - \hat{p}_m)/n_m}}
\]

\[
= \frac{(0.833 - 0.8 - 0.1)}{\sqrt{0.833 \times 0.17/150 + 0.8 \times 0.2/100}}
\]

\[
= -1.33
\]

-1.64, -1.65 What is your critical value when \(\alpha = 0.05\)?

0.09 What is your significance probability? (If necessary, give a bracket.)

In words pertinent to the problem, what conclusion do you reach (at the 0.05 level)?
Sadly, we cannot conclude that the Fox reporter is wrong. (To avoid any perception of sexism, let me assure everyone that in the other version of the exam, the null hypothesis is rejected.)

4. I want to argue that my 2016 FOCUS class is smarter than the average Duke student. Suppose the average Duke IQ is 120, and a random sample of 10 Focus students out of 17 had a mean IQ of 125 with a sample sd of 10.

In symbols, what is your alternative hypothesis? $H_A : \mu_F > 120$

2.39 What is the value of your test statistic?

The trick here is to use the FPCF. The test statistic is
\[
    t_s = \frac{\bar{X} - \mu_0}{(10/\sqrt{10}) \ast \sqrt{(17 - 10)/(17 - 1)}} = 2.39.
\]

1.83 For a 0.05 level test, what is your critical value?

This comes from a $t_9$ table.

0.025 to 0.02 What is my significance probability (if necessary, give a bracket).

Yes Do I decide my Focus class is smarter?

5. You want to argue that Duke students are smarter than students at UNC. To reduce variance, you control for major. You observe:

<table>
<thead>
<tr>
<th></th>
<th>Duke</th>
<th>UNC</th>
</tr>
</thead>
<tbody>
<tr>
<td>math</td>
<td>130</td>
<td>125</td>
</tr>
<tr>
<td>history</td>
<td>110</td>
<td>106</td>
</tr>
<tr>
<td>English</td>
<td>115</td>
<td>110</td>
</tr>
<tr>
<td>economics</td>
<td>120</td>
<td>114</td>
</tr>
<tr>
<td>statistics</td>
<td>150</td>
<td>145</td>
</tr>
</tbody>
</table>
In words pertinent to the problem, what is your alternative hypothesis?

The average IQ at Duke is larger than the average IQ at UNC.

TAs: Accept any equivalent wording.

15.81 What is the value of your test statistic?

This is a paired difference test. The differences are 5, 4, 5, 6, and 5, so the standard deviation of the differences is $sd_D = \sqrt{0.5}$. The mean for Duke is 125 and the mean for UNC is 120. So the test statistic is

$$ts = \frac{125 - 120}{\sqrt{0.5/5}} = 15.81.$$

2.13 For a 0.05 level test, what is your critical value?

It comes from a $t_4$ table.

In words pertinent to the problem, what conclusion do you reach (at the 0.05 level)?

You reject the null; Duke is smarter.

< 0.0005 What is your significance probability (if necessary, give bracketing values).