1. Benford’s Law says that in many cases, the first digit in a number (such as a street address) is not uniformly distributed but instead has probabilities as given in the following table:

<table>
<thead>
<tr>
<th>digit</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.30</td>
</tr>
<tr>
<td>2</td>
<td>0.18</td>
</tr>
<tr>
<td>3</td>
<td>0.13</td>
</tr>
<tr>
<td>4</td>
<td>0.10</td>
</tr>
<tr>
<td>5</td>
<td>0.08</td>
</tr>
<tr>
<td>6 or more</td>
<td>0.21</td>
</tr>
</tbody>
</table>

Check whether Benford’s Law holds for 1188 street addresses in Durham. The counts are:

<table>
<thead>
<tr>
<th>digit</th>
<th>probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>345</td>
</tr>
<tr>
<td>2</td>
<td>197</td>
</tr>
<tr>
<td>3</td>
<td>170</td>
</tr>
<tr>
<td>4</td>
<td>126</td>
</tr>
<tr>
<td>5</td>
<td>101</td>
</tr>
<tr>
<td>6 or more</td>
<td>249</td>
</tr>
</tbody>
</table>

In words, what is your alternative hypothesis?

Durham addresses do not follow Benford’s Law.

4.07 What is the value for your test statistic?

The expected values are $E_i = 1188 \cdot p_i$, so $E_1 = 1188 \cdot 0.3 = 356.4$, $E_2 = 213.48$, $E_3 = 154.44$, $E_4 = 118.8$, $E_5 = 95.04$ and $E_6 = 249.48$. The test statistic is

$$ts = \sum \frac{(O_i - E_i)^2}{E_i} = 4.07.$$  

11.07 What is your critical value for a 0.05 level test? (Give a number.)

The chi-squared value has 6-1=5 degrees of freedom.

P-value $> 0.25$ Give a bound or bounds for your P-value.

The test statistic lies between 2.343 and 7.289, so the P-value is between 0.8 and 0.2.

In words, what conclusion do you reach? (Use $\alpha = 0.05$.)

We fail to reject the null; we do not have reason think that Durham addresses do not follow Benford’s Law.

2. A random set of 100 professionals are classified according to handedness and job:

<table>
<thead>
<tr>
<th></th>
<th>left</th>
<th>right</th>
<th>ambidextrous</th>
</tr>
</thead>
<tbody>
<tr>
<td>CEO</td>
<td>10</td>
<td>18</td>
<td>2</td>
</tr>
<tr>
<td>statistician</td>
<td>15</td>
<td>30</td>
<td>5</td>
</tr>
<tr>
<td>ecldysiast</td>
<td>10</td>
<td>2</td>
<td>8</td>
</tr>
</tbody>
</table>
In words, what is your alternative hypothesis?

There is some relationship between job and handedness.

20.22 What is the value for your test statistic?

This is a test for independence. The expected values are the row sum times the column sum over the total, and then one sums \((\text{Obs} - \text{Exp})^2/\text{Exp}\) for all 9 cells.

9.49 What is your critical value for a 0.05 level test? (Give a number.)

This is a chi-squared distribution with 4 df.

In words, what conclusion do you reach?

Reject the null; there is some relationship between handedness and profession.