LAST NAME (Please Print): ______________________________

FIRST NAME (Please Print): ______________________________

HONOR PLEDGE (Please Sign): ______________________________

Statistics 111

**Homework 3**

You are allowed to discuss problems with other students, but the final answers must be your own work.

For all problems that require calculation, YOU MUST ATTACH SEPARATE PAGES, NEATLY WRITTEN, THAT SHOW YOUR WORK.

Please mark your answer in the space provided. As a general rule, each blank counts for one point. If necessary work is not shown, or if that work is substantially wrong, then you will not get credit even if the answer is correct. (The obvious purpose of this seemingly draconian policy is to prevent people from mindlessly copying each other's answers.)

Report all numerical answers to at least two correct decimal places.

**DUE DATE: START OF CLASS ON WEDNESDAY, SEPTEMBER 20.**
In this assignment, let $A$ be 1, 2 or 3, depending on whether your last name begins with A-F, G-S, or T-Z, respectively.

1. During World War II, the army tested inductees for syphilis. One approach is to give each man a test, but that is expensive. Alternatively, you can divide each man’s blood sample into two parts. Then one pools the half-samples from $n$ men and tests it for syphilis. If the test is positive, one goes back and tests each of the samples separately; if the test is negative, one need not. Suppose the rate of syphilis is 3%.

What is the expected number of tests one would need if $n = A + 3$?

What is the optimal $n$?

2. Let $X$ and $Y$ have constant joint density on the interior of the triangle with vertices at $(0,0)$, $(3,0)$, and $(0, 3)$.

Are $X$ and $Y$ independent? Explain. (2 pts)

Find the marginal densities of $X$ and $Y$.

$$f_1(x) =$$

$$g_2(y|x) =$$
3. You buy a stock for $X$ dollars, and sell it for $Y$ dollars. The joint density is

\[ f(x, y) = \begin{cases} \frac{x}{36} & \text{for } 0 < x < y < 6, \\ 0 & \text{otherwise.} \end{cases} \]

What is your expected profit?

What is the marginal density of $Y$?

\[ f_2(y) = \]

What is the conditional density of $X$ given $Y = y$?

\[ g_1(x|y) = \]

What is the covariance between $X$ and $Y$?

4. An ice cream stand sells three flavors: chocolate, vanilla and bacon. Suppose the popularity of chocolate is $p_1$ and the popularity of vanilla is $p_2$. Find a formula for the probability that among $n$ children, $x_1$ will choose chocolate, $x_2$ will choose vanilla, and the rest will choose bacon.

\[ p(x_1, x_2, x_3) = \]
5. Let $X$ and $Y$ have joint density $f(x, y) = 14 \exp(-2x - 7y)$ for $x > 0$ and $y > 0$, and $f(x, y) = 0$ otherwise. What is $\Pr[X > Y]$?

6. Let $X$ have density $f(x) = kx^2(1-x)^2$ for $0 \leq x \leq 1$, and it is zero otherwise.

- What is $k$?
- What is $\Pr[X \geq 3/4]$?
- What is $E[X]$?
- What is $\text{Var}[X]$?
- What is the standard deviation of $X$?

How are you holding up? Approximately how long did this homework take you?