

LAST NAME (Please Print): _____

FIRST NAME (Please Print): _____

HONOR PLEDGE (Please Sign): _____

Statistics 111

Midterm 1

- This is a closed book exam.
- You may use your calculator and a single page of notes.
- The room is crowded. Please be careful to look only at your own exam. Try to sit one seat apart; the proctors may ask you to randomize your seating a bit.
- Report all numerical answers to at least two correct decimal places or (when appropriate) write them as a fraction.
- All question parts count for 1 point.

1. Governor McCrory is told that statistics majors are less likely to get jobs after graduation than English majors. To support his North Carolina jobs program, he plans to shut down the state's statistics departments. The data are as follows:

	Duke			NCSU			UNC	
	Job	No Job		Job	No Job		Job	No Job
English	50	30		1	19		3	37
Statistics	15	5		30	270		6	54

_____ What proportion of English majors have jobs?

_____ What proportion of statistics majors have jobs?

_____ At Duke, what proportion of English majors have jobs?

_____ At Duke, what proportion of statistics majors have jobs?

_____ What is the adjusted proportion of English majors who have jobs?

This is an example of Simpson's Paradox. How would you explain what is happening to Governor McCrory?

_____ 2. A statistics final consists of 80 multiple-choice questions. You are a clueless student, and answer at random. Your chance of guessing correctly is $1/5$. What is the approximate probability that you correctly answer more than 18 questions?

_____ 3. In an average month, 2.6 celebrities get married. What is the probability that three or more celebrities get married next month?

4. Consider the density function $f(x) = cx^2(2 - x)$ on the interval $[0, 2]$.

_____ What is c ?

_____ To avoid propagation of error, assume $f(x) = 12x^2(1 - x)$ on $[0, 1]$. What is $\mathbb{E}[X]$?

_____ For the density above, what is the standard deviation of X ?

5. Consider the following numbers (a sample from a population):

0, 0, 9, -10, 7, 8, 4, 2

_____ What is the IQR? _____ What is the mode?

_____ What is the variance?

_____ Suppose a sample has standard deviation equal to 4. You add 3 to each number and multiply by -2. What is the new standard deviation?

_____ Suppose a set of numbers has median equal to 4. You add 3 to each number and multiply by -2. What is the new median?

6. A class has 24 students—three males and three females from each college year.

_____ You draw a student at random. What is the probability that it is a female or a senior?

_____ You draw three times, without replacement. What is the probability that all three are male?

_____ You draw three times without replacement. What is the probability that the last draw is a female if the first two draws were male?

7. Let $f(x, y) = 2$ for $0 < y < x < 1$.

What is the marginal density of X ? Indicate the support.

What is the conditional density of Y given $X = x$? Indicate the support.

_____ What is the probability that $0 < Y < 0.3$ given that X is 0.5?

_____ 8. A university has 15 residence halls, all of equal size. If 6 students are drawn at random, what is the probability that two or more are in the same residence hall?

9. You toss a coin that has probability $1/3$ of coming up Heads. If it comes up Heads you draw a random value from an exponential distribution with $\lambda_1 = 1.5$. If it comes up tails you draw a random value from an exponential with $\lambda_2 = 3.5$.

_____ What is the expected value of your random variable?

_____ If you observe $X > \frac{1}{2}$, what is the probability that you threw Tails?

_____ 10. Suppose 10% of students at Duke are from Virginia, suppose 48% are male, and suppose 30% have GPAs greater than 3.2. Under independence, what is the probability of sampling a student who is Virginian, or male, or has GPA greater than 3.2?

_____ 11. You roll a fair die 10 times. Let X be the number of sixes in the first four rolls, and let Y be the total number of sixes. What is $\text{Cov}(X, Y)$?

12. The annual average temperature in July in Durham is normally distributed with mean 87 degrees and standard deviation 3 degrees. A climate scientist warns that global warming will raise the mean by 2 degrees in 2020, and you decide to believe him if the average temperature in July of 2020 is 91 or higher. (To make this a realistic question, assume you have a time machine at your disposal.)

_____ If the climate scientist is wrong, what is the probability that you mistakenly believe him?

_____ If the climate scientist is correct, what is the probability that you believe him?

13. Suppose $f(x, y) = c$ for $(x - 3)^2 + (y - 5)^2 \leq 4$ and it is zero elsewhere.

_____ What is c ?

_____ Are X and Y independent?

What is $f_1(x)$? Indicate the support.

_____ What is $\mathbb{E}[X]$?

What is $\text{Var}[X]$? Write as an integral. _____

What is $g_2(y|x)$? Indicate the support.

_____ What is the covariance between X and Y ?